



Hashemite University

Mechatronics Engineering Department

Automatic Control Laboratory

Manual



Experiment 1

Speed Control System

Objectives:

- To analyze the response of a first order system.
- To identify the constants in the mathematical models of a DC motor and a tachometer experimentally.
- To construct various type of dc motor speed control system.

Apparatus:

Servo kit components: (see figure 1.1)

1. Servo amplifier : SA150D
2. Servo motor :DCM150F
3. Power supply : PS150E
4. Attenuator unit : AU150B
5. Operational amplifier : OU150A
6. Tachometer : GT150X
7. Voltmeter – Oscilloscope – Function generator

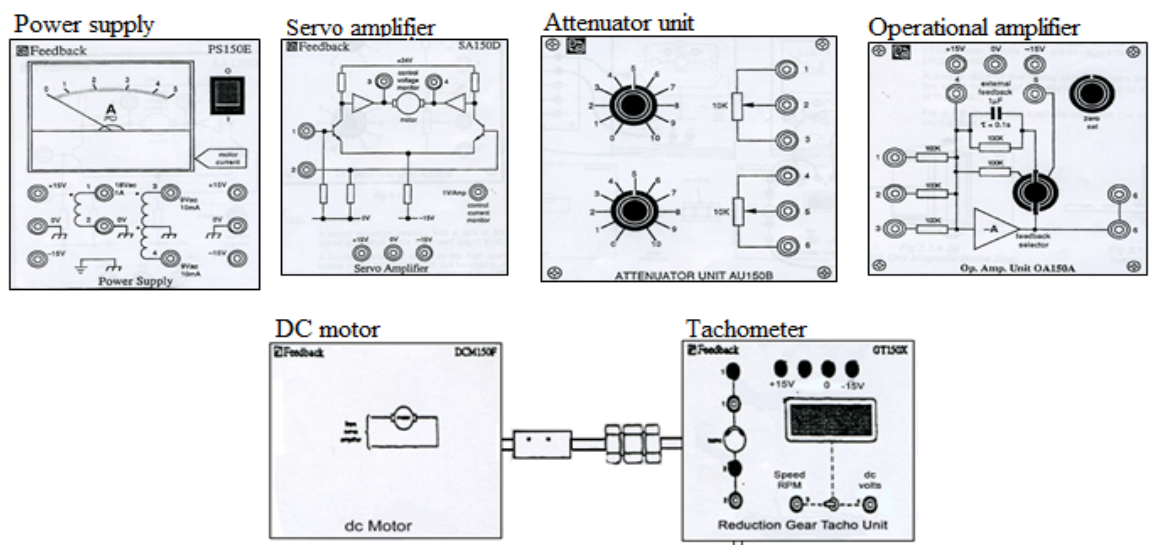


Figure 1.1: Servo Motor System Components

Theoretical Background:

DC motors are used extensively in many control applications. Therefore, it is necessary to establish mathematical models for DC motors. The transfer function of a DC motor can be approximated by a first order model with unknown constants. These constants can be identified experimentally.

The tachometer works essentially as a voltage generator, with the output voltage a proportional to the magnitude of the angular velocity of the input shaft. Tachometers are used in control systems in many ways; they can be used as a speed indicator to provide shaft-speed readout or to provide speed feedback signal in the case of speed control systems.

System components:

1. Servo amplifier [SA150D]:

This unit operates the motor from signals applied to the input sockets 1 or 2.

These two inputs allow reversing the motor rotation. The servo amplifier is connected to the servo motor by a plug and cable. The gain of this unit $K_{\text{servo}} = 7.4$ Terminals for ± 15 volts and ground (common) are variable on this unit.

2. power supply [PS150E]:

This unit provides the various supplied required for the servo components. There are terminals for ± 15 volts, and common (ground). An ammeter is also included. The maximum current is 2 A. A socket and cable connects this unit to the servo amplifier.

3. Attenuator unit [AU150B]:

This unit consists of two separate 10 K Ω potentiometers. Each potentiometer can provide a gain between (0-1) depending on the position of its knob. This unit can be used as either attenuator or controller.

4. Operational amplifier unit [OU150A]:

This unit is an operational amplifier. A selector switch allows the use of the unit as a summer, integrator or with any external circuit in the feedback path. The unit is used as an error detector which determines the difference between the demand and response. Before using the unit, measure the output of the unit and adjust it to zero with the zero control, when the input voltage at sockets 1 and 2 are zero.

Depending on the position of its knob this unit can be modeled as

1. Summation point of unity gain $k_{\text{op}}=1$

2. Summation point with transfer function: $T_{op}(s) = \frac{1}{1+s}$
3. Summation point with transfer function depending on an external circuit.

5. Servo motor [DCM150F]:

This unit is a DC motor that produces a torque. The tachometer with terminal +, - and common (ground) is attached to the motor. The transfer function of the motor is:

$$\frac{\Omega(s)}{E(s)} = km \frac{1}{1+s\tau_m} \quad (1.1)$$

Where:

$\Omega(s)$: output speed Laplace transform

$E(s)$: input voltage Laplace transform

k_m : motor torque constant ([rad/sec] / volts)

τ_m : motor time constant (sec)

If the input voltage is a step, then the output speed (with zero initial condition) is:

$$\omega(t) = k_m (1 - e^{-\frac{t}{\tau_m}}) \quad (1.2)$$

Response Analysis:

For a permanent magnet dc motor, and assuming the armature inductance $L_a=0$, the transfer function is represented in equation (1.1). It is clear that the system is a first order system with a time constant τ_m and a dc gain k_m . Figure 1.2 present the step response of a first order system.

The system parameter (the time constant and the dc gain) can be obtained experimentally from the response. The dc gain is the ratio between the final value of the output and the input. Whereas the time constant is the time at which the output reaches 63% of its final value.

Many systems can be modeled as a first order system by the following general form differential equation:

$$\frac{1}{b} \dot{y} + \frac{a}{b} y = r(t) \quad (1.3)$$

Taking its Laplace transformation, the following transfer function is obtained:

$$\frac{Y(s)}{R(s)} = \frac{b}{s+a} \quad (1.4)$$

The time domain response of system to a unit step input is given by Figure 1.2 and equation 1.5.

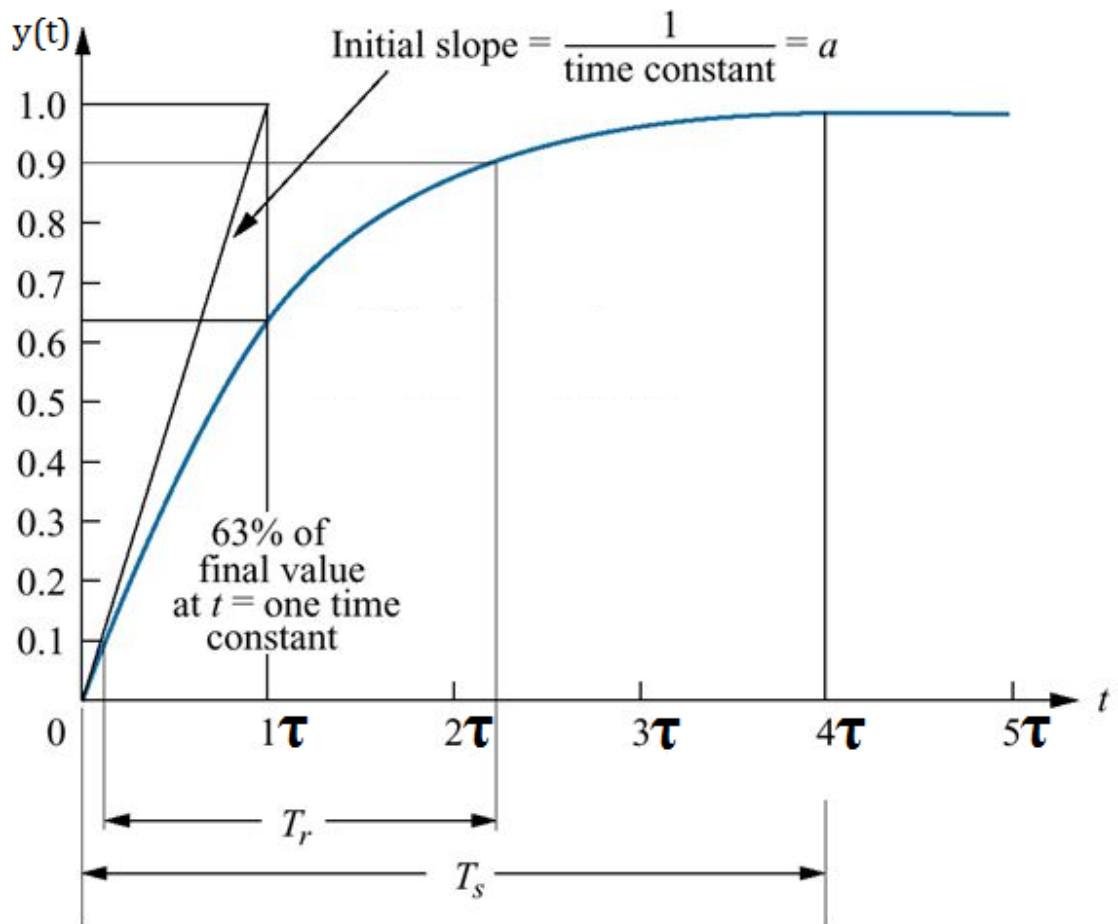


Figure 1.2: Response of a first order system to a unit step input

$$y(t) = b(1 - e^{-at}) \quad (1.5)$$

The characteristics of the system can be defined with help of Figure 1.2 by the following parameters:

1. System gain (b): the system gain (b) can be defined as the dc value (steady state value of the output $y(t)$) and can be given by:

$$b = \lim_{s \rightarrow 0} sY(s) = \lim_{s \rightarrow 0} \left(\frac{1}{s} \frac{b}{s+a} \right) = \lim_{t \rightarrow \infty} y(t) \quad (1.6)$$

or from Figure 1.2,

$$b = \frac{y_{\text{steady-state}}}{r_{\text{steady-state}}} \quad (1.7)$$

2. Time constant (τ): the time constant ($1/a$) is defined as time value when the system reaches 63% of its steady-state value. And can be found by the two methods illustrated by Figure 1.2.

$$\tau = \frac{1}{a} \quad (1.8)$$

3. Rise time (T_r): the rise time (T_r) is defined as the time taken by the system to change from 10% till 90% of its steady-state value.

$$T_r = \frac{2.2}{a} \quad (1.9)$$

4. Settling time (T_s): the rise time (T_s) is defined as the time taken by the system to reach 98% of its steady-state value.

$$T_s = \frac{4}{a} \quad (1.10)$$

Procedure:

The experiment consists of two parts: open and closed loop speed control systems.

1. Open Loop System

a. Measurement of tachometer coefficient [k_t]

The tachometer output voltage v_t is proportional to the output speed, i.e. $v_t = k_t \omega$, where tachometer constant. The following procedure can be used to identify the tachometer constant.

1. Zero set the OP-AMP, (i.e. the input voltage to op-amp must be zero and measure the output V_{out} , turn the knob until $V_{out} = 0$).
2. Connect the circuit shown in Figure 1.3 (Open Loop Speed Control).
3. Slowly increase V_{in} till the motor just start rotate, this voltage is due brush friction and it will be offset from zero when no speed.
4. Fill in Table 1.1.

Table 1.1

V_{in}	V_{out} (Tacho)	Speed
0		
0.5		
1		

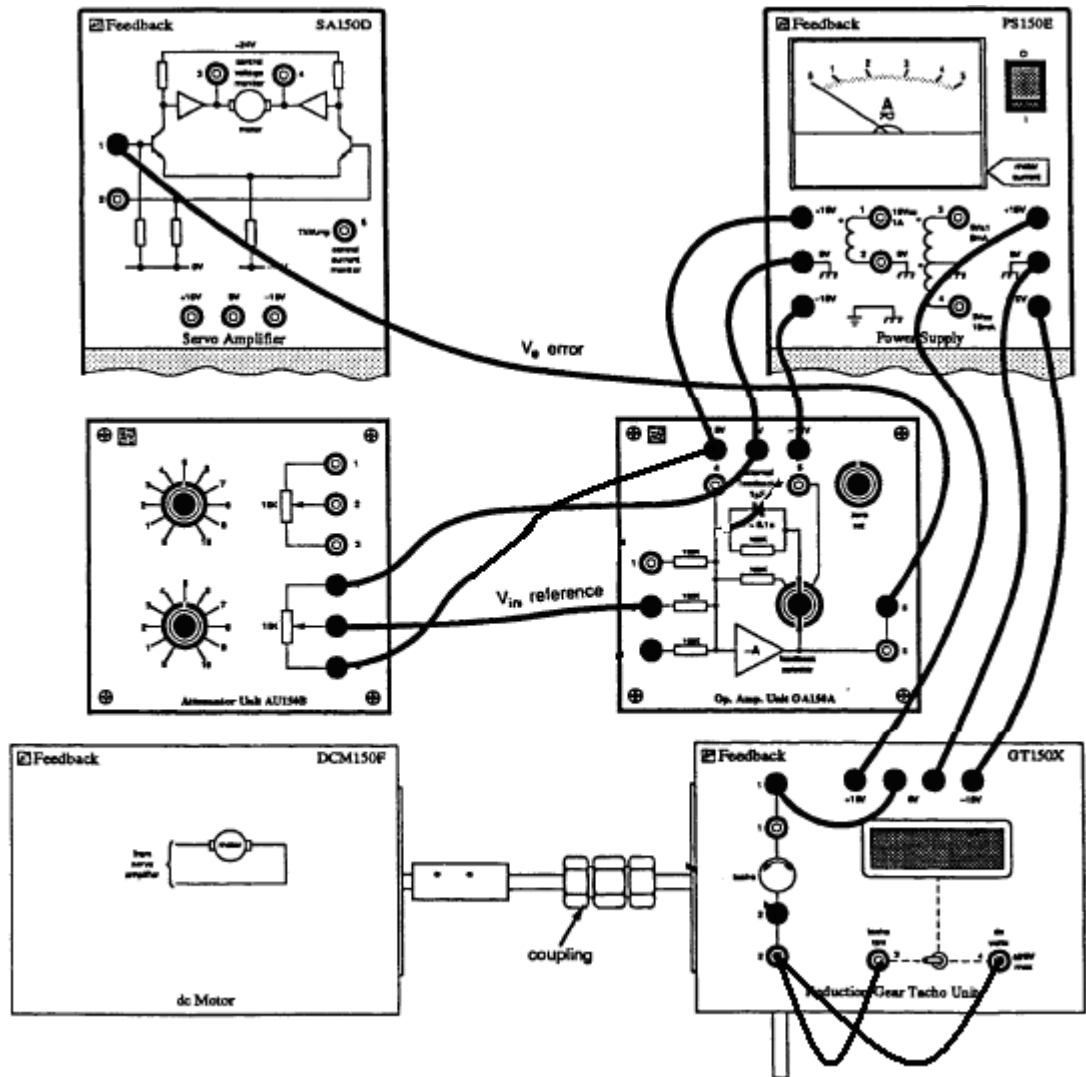


Figure 1.3: Open loop System with Variable Input

b. Measurement of the servo motor constant k_m and measurement of the motor time constant τ_m

1. Zero set the OP-AMP, (i.e. the input voltage to op-amp must be zero and measure the output V_{out} , turn the knob until $V_{out} = 0$).
2. Connect the circuit shown in Figure 1.4 (Open Loop Speed Control).
3. Set the function generator to provide 0.2 Hz square wave with 4 step input.
4. Plot a figure of tachometer voltage vs. operational amplifier input voltage by using the oscilloscope, connect channel 1 to the operational amplifier input voltage and connect channel 2 to the tachometer voltage then invert channel 2.
5. From the figure of step 4 find the time constant (τ_m).
6. Calculate the value of k_m . Clearly show how you find this value.

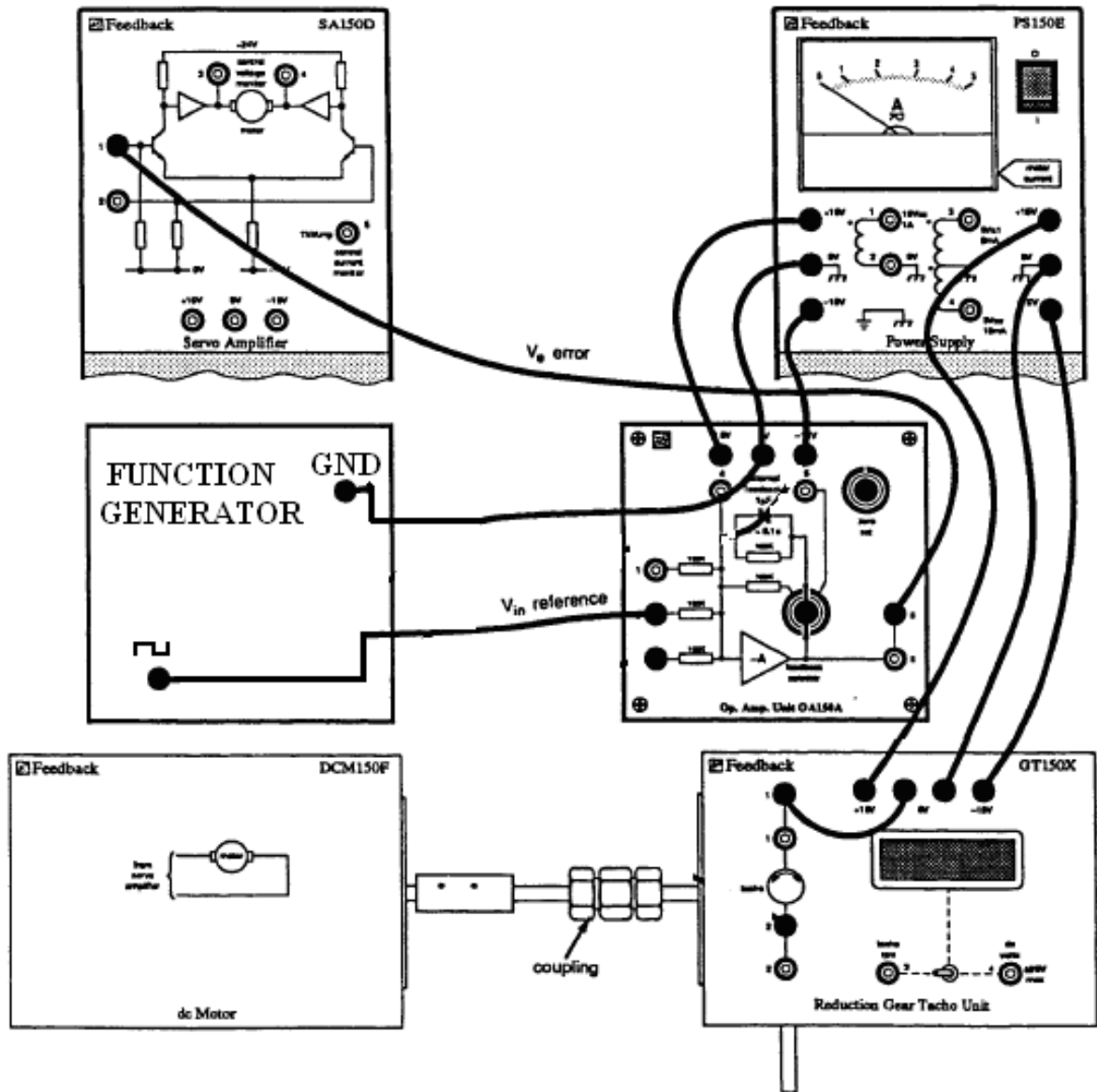


Figure 1.4: Open Loop System with Step Input

2. Closed Loop System

a. Measurement of the motor time constant (τ_m) for closed loop system without magnetic brake.

1. Zero set the OP-AMP, (i.e. the input voltage to op-amp must be zero and measure the output V_{out} , turn the knob until $V_{out} = 0$).
2. Connect the circuit shown in Figure 1.5 (closed Loop Speed Control).

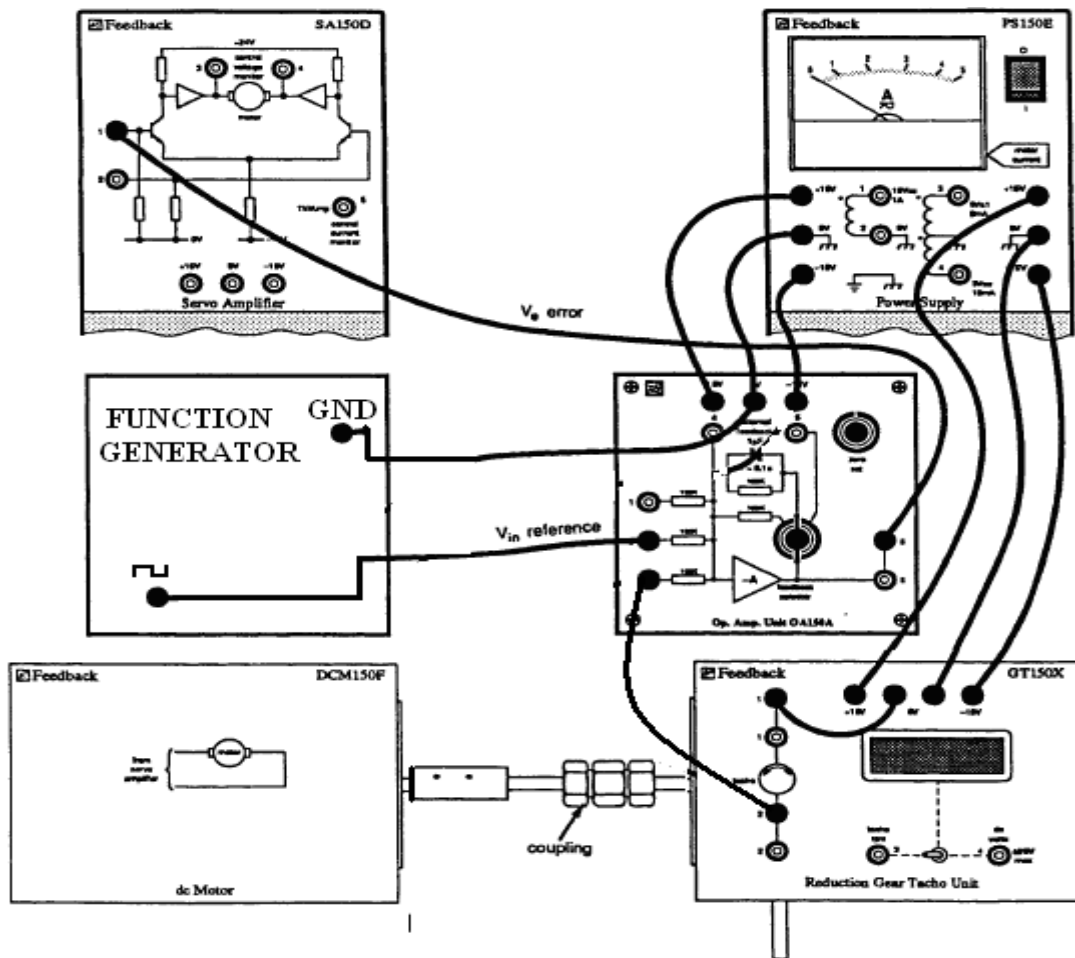


Figure 1.5: Closed loop system with step input

3. Set the function generator to provide 0.2 Hz square wave with 4 step input.
4. Plot a figure of tachometer voltage vs. operational amplifier input voltage by using the oscilloscope, connect channel 1 to the operational amplifier input voltage and connect channel 2 to the tachometer voltage then invert channel 2.
5. From the figure of step 4 find the time constant.

Discussion and Analysis:

1. Explain the behavior of the motor speed when the input voltage is increased.
2. Compare the closed loop system with the open loop system.
3. Calculate the value of K_t .

Experiment 2

Position Control System

Objectives:

After you finish this experiment, you are expected to:

- Have established the operating characteristic of the pre-amplifier.
- Have observed the operation of a basic open-loop position control system.
- Have observed the response of closed loop position control system to a step input.

Before working the assignment, we have to look at the pre-amplifier, and tacho unit.

- To evaluate the performance of a second order system. The step response test use to investigate the accuracy and the transient behavior of an under damped second order system.

Apparatus:

Servo kit components used to control the position of the dc motor are the same used in the last experiment in addition to the components in figure 2.1

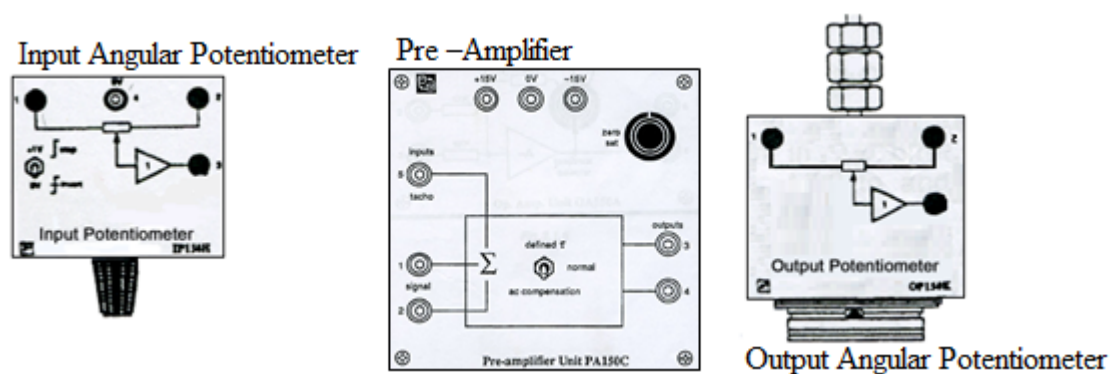


Figure 2.1: position control system components

1. Pre –Amplifier Unit [PA150C].
2. Output Angular Potentiometer.
3. Input Angular Potentiometer.

Theoretical Background:

DC motors are used extensively in many control applications. In some applications - such as robot arm, machine tools, and valves- the position of motor is the output of interest rather than the speed.

System components:

The same components are used as in the previous experiment (speed control system) with the addition of the pre amplifier circuit. The model of the DC motor in terms of its position becomes a second order system.

1. Pre –Amplifier Unit [PA150C]:

This unit has two inputs are effectively summed, allowing two signals to be applied.

A positive signal applied to either input causes the upper output (3) to go positive, the output (4) staying constant value near ground.

A negative input causes the lower output (4) to go positive, the output (3) staying constant value near ground.

2. Output Angular Potentiometer

This unit is connected to Tachometer unit [GT150X]. It gives an output voltage proportional to the angular position of the motor after a reduction gearbox with a ratio of 30/1.

Response Analysis

For a permanent magnet dc motor, and assuming the armature inductance $L_a=0$, the transfer function is represented in equation (2.1).

$$\frac{\theta(s)}{E(s)} = \frac{k_m}{s(\tau_m s + 1)} \quad (2.1)$$

Where:

$\theta(s)$: output speed Laplace transform

$E(s)$: input voltage Laplace transform

k_m and τ_m are respectively the motor torque constant and time constant obtained in the last experiment.

It is clear that the system is a second order system.

The standard form of the transfer function of a second order system is given by the following equation

$$T(s) = K \frac{\omega_n^2}{s^2 + 2\omega_n\zeta s + \omega_n^2} \quad (2.2)$$

Where:

ω_n is the natural frequency

ζ is the damping ratio

K is the Dcgain

If the input of the system is a unit step, then,

$$Y(s) = K \frac{\omega_n^2}{s(s^2 + 2\omega_n\zeta s + \omega_n^2)} \quad (2.3a)$$

Taking the Laplace inverse of equation 2.3a

$$y(t) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_n \sqrt{1-\xi^2} * t + \cos^{-1} \xi) \quad (2.3b)$$

The characteristic equation of this system is;

$$s(s^2 + 2\zeta\omega_n s + \omega_n^2) = 0 \quad (2.4)$$

$s = 0$ pole at origin or $s^2 + 2\omega_n\zeta s + \omega_n^2 = 0$ then;

$$s_{1,2} = -\zeta\omega_n \pm \omega_n \sqrt{\zeta^2 - 1} \quad (2.5)$$

From equation 2.5, the two poles of the system might be

$$\text{two poles} \begin{cases} 1. \text{ complex conjugates} : \zeta < 1 \text{ (underdamped)} \\ 2. \text{ real equal} : \zeta = 1 \text{ (critically damped)} \\ 3. \text{ real distinct} : \zeta > 1 \text{ (overdamped)} \end{cases}$$

Figure 2.2 shows the step response for different values of ζ . Note that the less the damping ratio, the more the system oscillation.

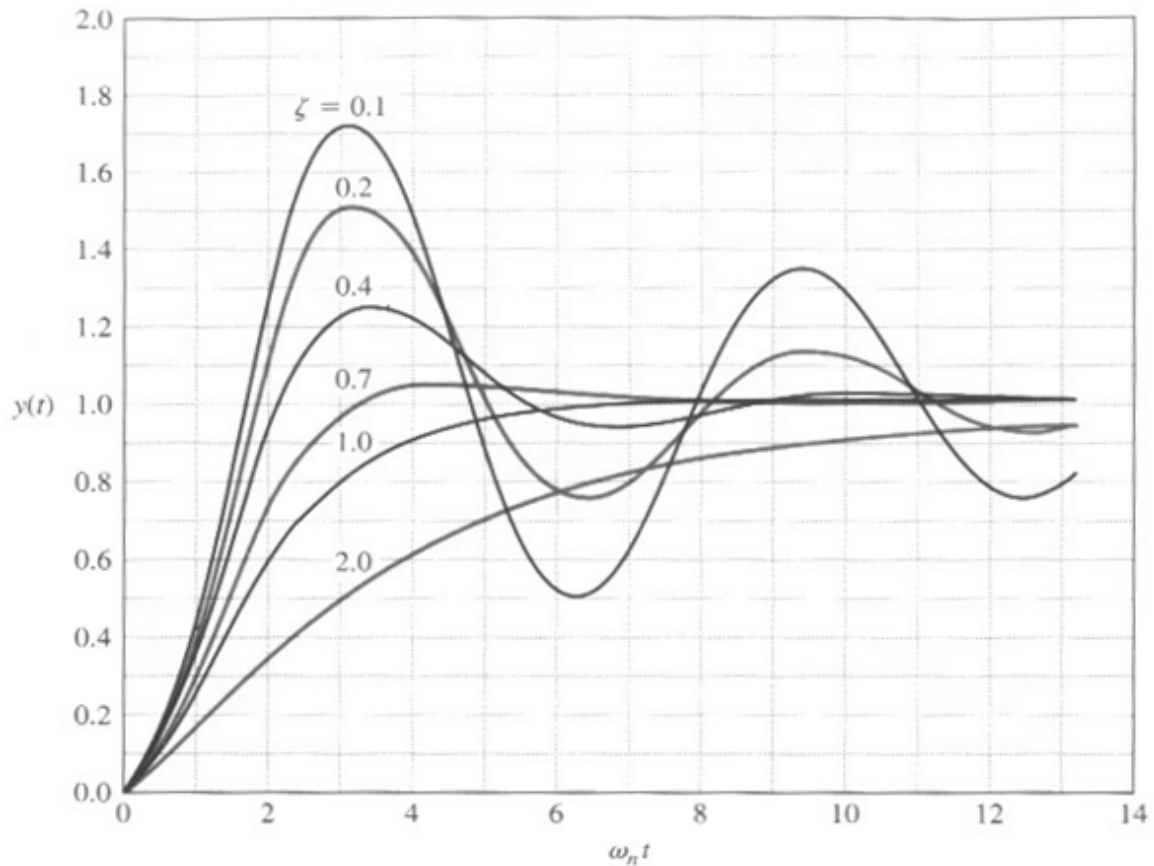


Figure 2.2: The Step Response for Different Values of ζ

For an under damped system $0 < \zeta < 1$

$$s_{1,2} = \underbrace{-\zeta\omega_n}_{\text{Real part}} \pm \underbrace{\omega_n \sqrt{\zeta^2 - 1}}_{\text{Imaginary part}}$$

The under damped response the system and some of the performance specifications are illustrated by Figure 2.3. These system specification are given by the following equations

Percent overshoot

$$\text{PO \%} = 100 e^{-\xi\pi / \sqrt{1-\xi^2}} \quad (2.6)$$

Peak time

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} \quad (2.7)$$

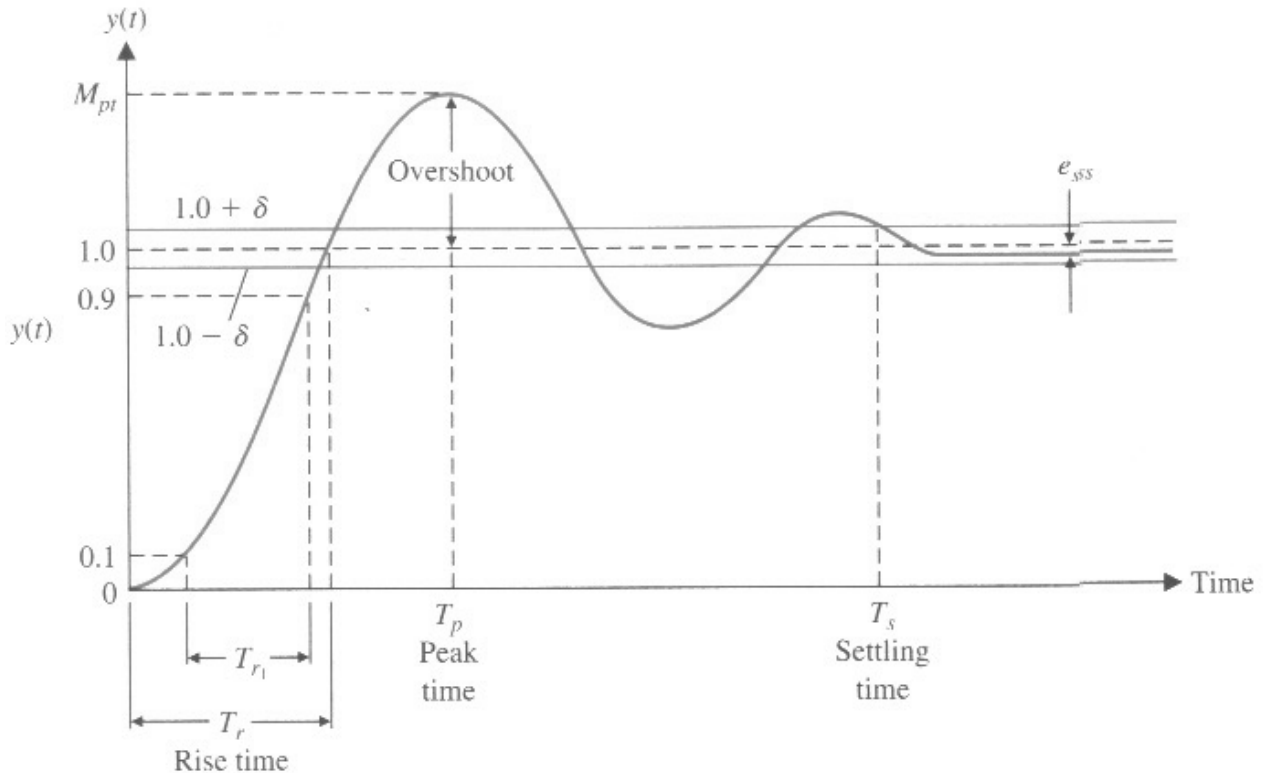


Figure 2.3: The Unit Step Response of An Under Damped System

Settling time (2% error)

$$T_s = \frac{4}{\xi \omega_n} \quad (2.8)$$

Rise time

$$T_{r1} = \frac{2.16\xi + 0.64}{\omega_n} \quad (2.9)$$

Note that all system specifications are functions of the system natural frequency and damping ratio which are both functions of the physical system parameter such as R,L and C for electrical systems or M,K, and B for mechanical systems.

Procedure:

The experiment consists of two parts: open and closed loop speed control systems.

1. Characteristic of the pre-amplifier :

In pre-amplifier before you make the experiment, you must select the normal condition by using toggle switch.

1. Connect the circuit shown in Figure 2.4 (Characteristic of the pre-amplifier).
2. Zero set the preamplifier. This should be achieved by switching the "zero set" knob until $\Delta V_{34}=0$.
3. For scale position on the potentiometer, take readings with the voltmeter of each of the pre-amplifier unit outputs with respect to GND.
4. Fill in Table 2.1.

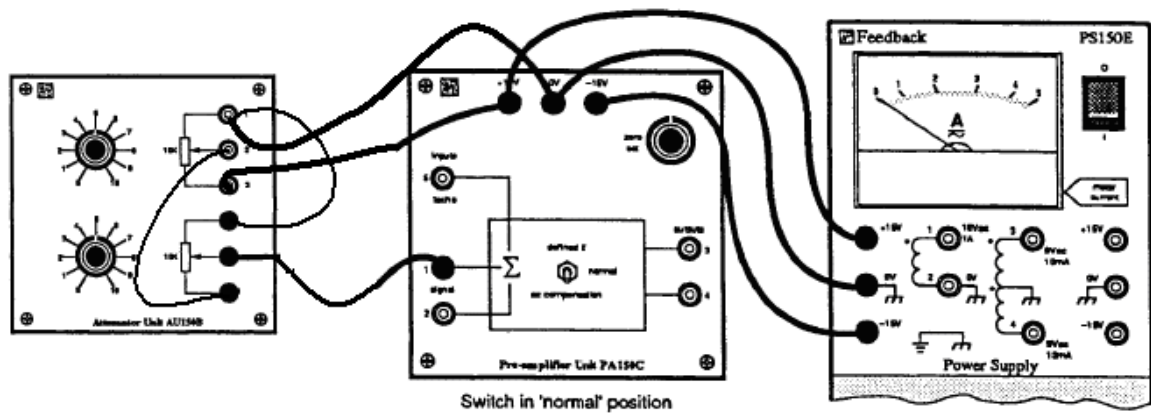


Figure 2.4: Characteristic of the pre-amplifier

Table 2.1

Input Signal	Pre-amplifier output		
V_i (Volts)	$V_o(3)$	$V_o(4)$	$V_o(4)- V_o(3)$
0.1			
0.2			
0.3			
0.4			
0.5			
0.6			
0.7			

5. Connect terminal 3 of the potentiometer to $-15V$.
6. Fill in Table 2.2.

Table 2.2

Input Signal	Pre-amplifier output		
	V_i (Volts)	$V_o(3)$	$V_o(4) - V_o(3)$
-0.1			
-0.2			
-0.3			
-0.4			
-0.5			
-0.6			
-0.7			
-0.8			
-0.9			

2. Position control

a. Determination of output potentiometer gain $[k_o]$:

1. Connect the electrical circuit as shown in figure 2.5.
2. Set the output potentiometers to 0° .
3. Fill Table 2.3.

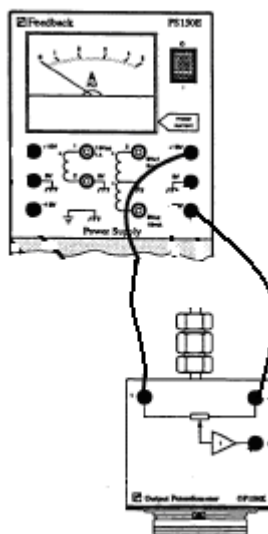


Figure 2.5: Output potentiometer gain

Table 2.3

Output potentiometer	V_{out}
0°	
10°	
20°	
30°	
40°	
50°	
60°	
70°	
80°	
90°	
100°	
110°	
120°	
130°	
140°	
150°	

b. Open loop system

1. Zero set the OP-AMP
2. Connect the servo trainer kit to build an open loop position control system as shown Figure 2.6.
3. Set the function generator to provide 0.2 Hz square wave with 4 step input.
4. Plot the output potentiometer voltage vs. operational amplifier input voltage by using the oscilloscope, connect channel 1 to the operational amplifier input voltage and connect channel 2 to the output potentiometer voltage then invert channel 2.

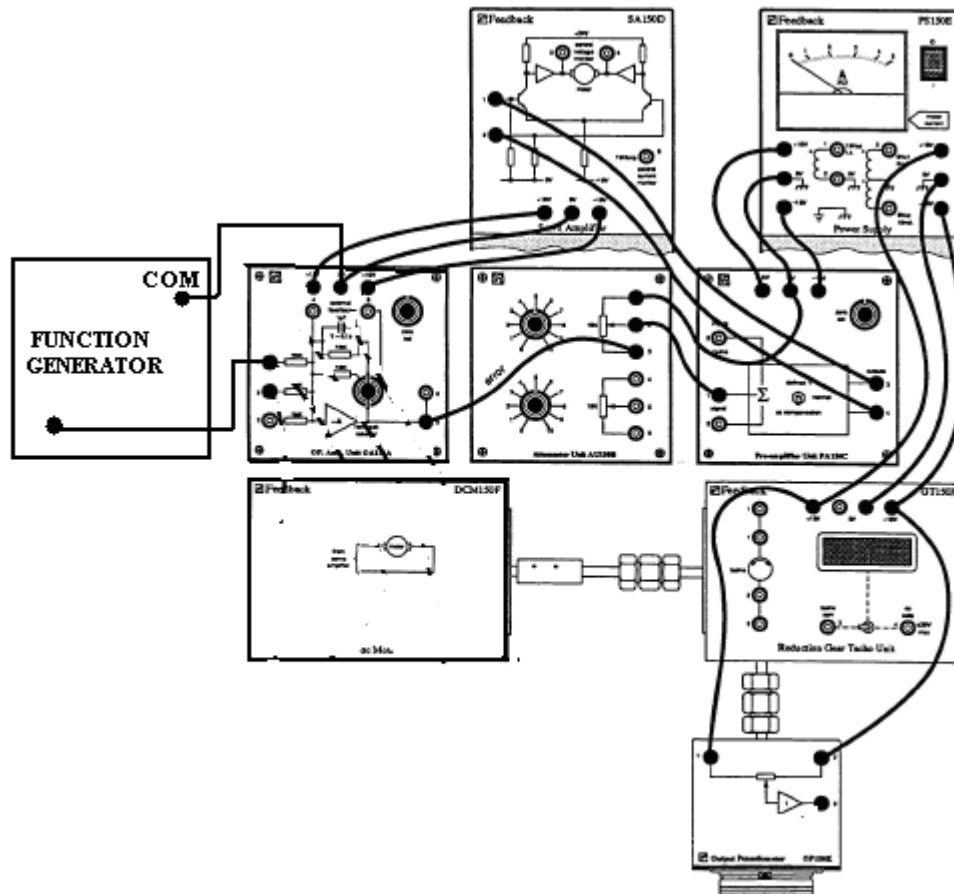


Figure 2.6: Open Loop Position Control System

c. closed loop system

1. Zero set the OP-AMP.
2. Connect the servo trainer kit to build a closed loop position control system as shown in Figure 2.7.
3. Set the function generator to provide 0.2 Hz square wave with 4 step input.
4. Plot the output potentiometer voltage vs. operational amplifier input voltage by using the oscilloscope, connect channel 1 to the operational amplifier input voltage and connect channel 2 to the output potentiometer voltage then invert channel 2.
5. Repeat step 4 for different values of controller gain (attenuator).

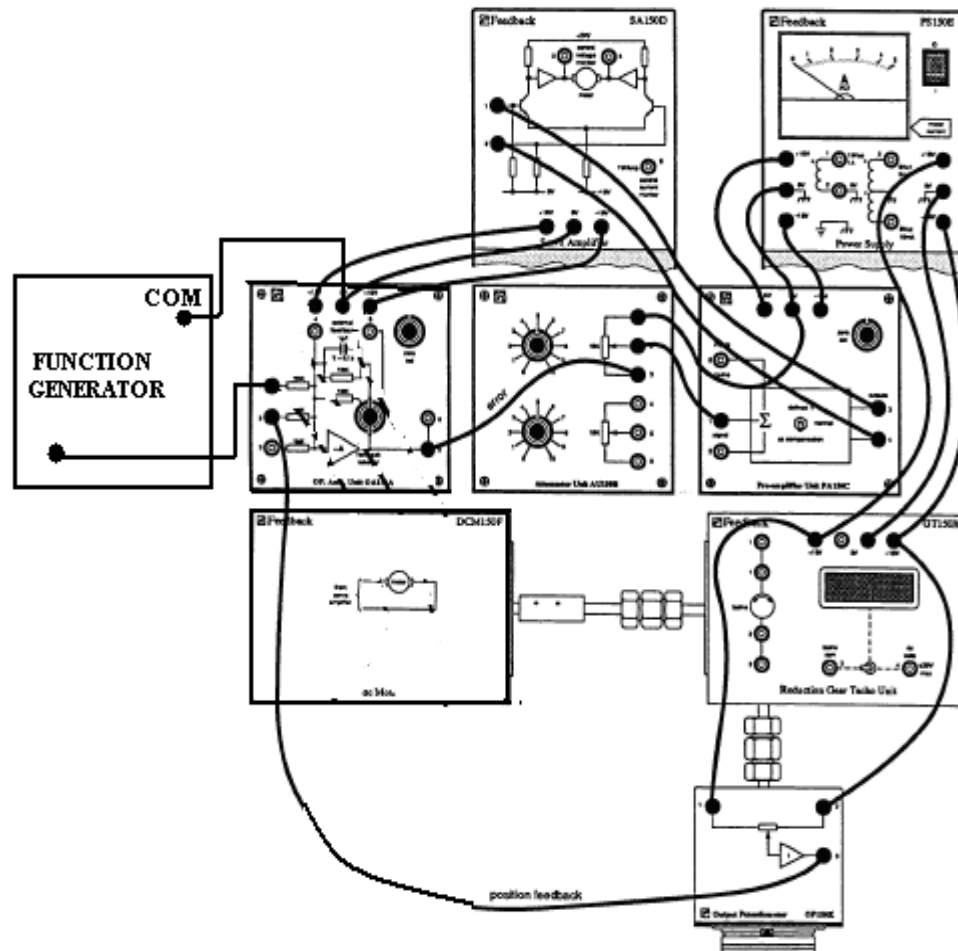


Figure 2.7: Closed Loop system Position Control

Discussion and Analysis:

1. Plot graph of the input volts against the output volts for $V_o(3)$ & $V_o(4)$ on the same graph.
2. Plot graph of the output $V_o(4-3)$ against the input V_i to get the gain K .
3. Compare the closed loop system with the open loop system.

Experiment 3

Characteristics of Open-Loop and Closed Loop Systems

Objectives:

- To investigate the characteristics of an open-loop system. Sensitivity, accuracy, disturbance rejection, and transient response are considered in this experiment.
- To evaluate the performance of a closed-loop system in comparing with that of the open-loop system. The performance characteristics to be considered include:
 - Disturbance rejection
 - Sensitivity to variation in forward-path gain
 - Accuracy
 - Closed-loop system disturbance rejection improvement
 - The extent to which loop gain affects these performance characteristics

Apparatus:

Servo kit components: (see figure 3.1)

1. Servo amplifier : SA150D
2. Servo motor :DCM150F
3. Power supply : PS150E
4. Attenuator unit : AU150B
5. Operational amplifier : OU150A
6. Tachometer : GT150X
7. Voltmeter – Oscilloscope – Function generator

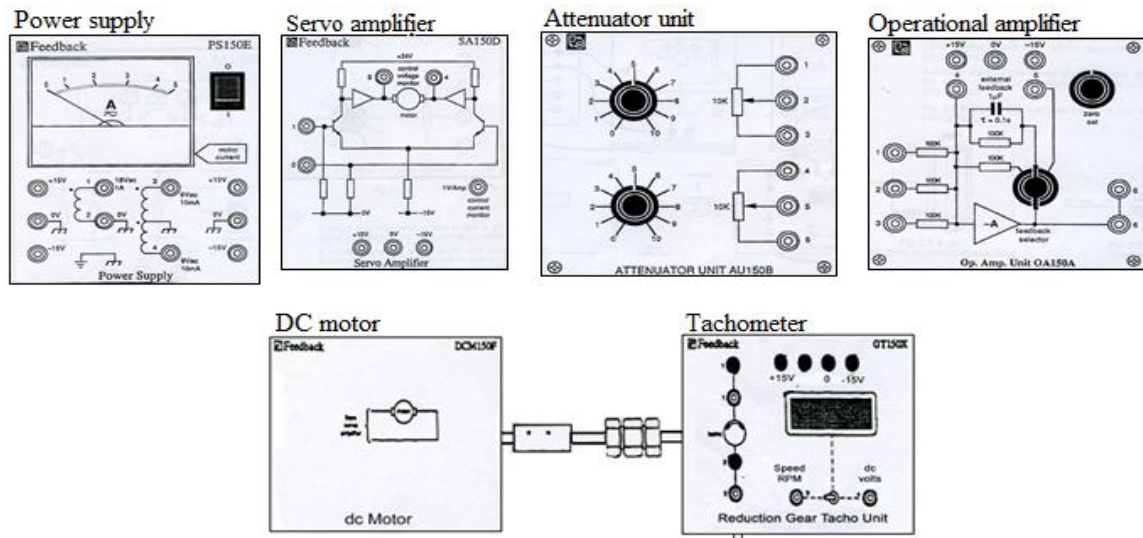


Figure 3.1: Servo Motor System Components

Theoretical Background:

A control system is an interconnection of components forming a system configuration that will provide a desired system response. The basis for analysis of a system is the foundation provided by linear system theory, which assumes a cause effect relationship for the components of a system. Therefore a component or process to be controlled can be represented by a block as shown in Figure 3.2. the input-output relationship represents the cause-and-effect relationship of the process, which in turn represents a processing of the input signal to provide an output signal variable, often with power amplification.

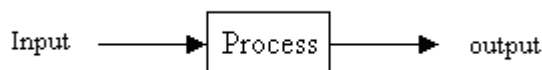


Figure 3.2: Process to Be Controlled.

An open loop system uses a controller and an actuator to obtain the desired response, and it is a system without feedback as in Figure 3.3.

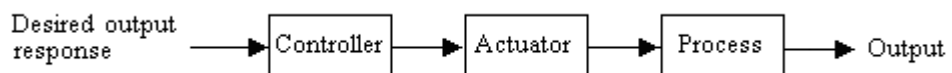


Figure 3.3: Open-loop Control System

In contrast a closed-loop system utilizes an additional measure of the actual output to compare the actual output with the desired output response. The measure of the output is called feedback signal. A feedback control system is a control system that tends to maintain a prescribed relationship of one system variable to another by comparing functions of these variables and using the difference as a means of control. See Figure 3.4.

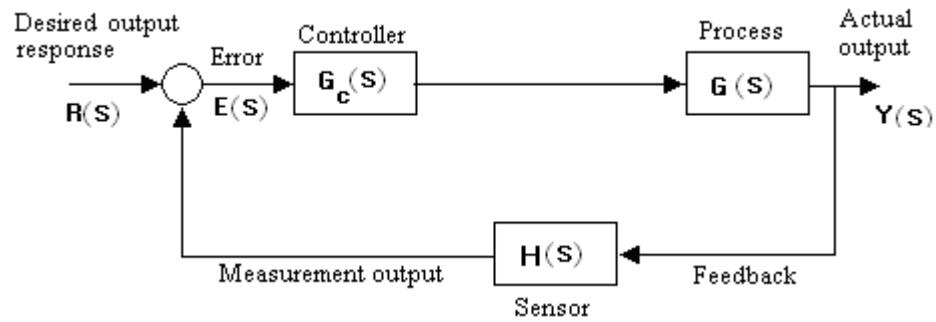


Figure 3.4: Closed-loop feedback control system

Control systems performance is judged by several characteristics such as Accuracy and steady-state error, sensitivity and disturbance rejection.

The accuracy of a system can be studied by observing the error in the system. From the system block diagram in Figure 3.4, the error is

$$E(s) = R(s) - Y(s)H(s) \quad (3.1)$$

Assuming unity feedback

$$E(s) = R(s) - Y(s) \quad (3.2)$$

Then

$$E(s) = \frac{1}{1 + G_c G(s)} R(s) \quad (3.3)$$

The smaller the error, the higher the accuracy of the system. The steady-state error of the system is

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) \quad (3.4)$$

Another important characteristic of control systems is system sensitivity. System sensitivity is the ratio of the changes in the system transfer function to the change of the process transfer function (or parameter) for a small incremental change.

System transfer function is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} \quad (3.5)$$

Therefore, the sensitivity is defined as:

$$S_G^T = \frac{\Delta T(s)/T(s)}{\Delta G(s)/G(s)} \quad (3.6)$$

In the limit, for small incremental changes, Equation 3.6 becomes:

$$S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{\partial T}{\partial G} \cdot \frac{G}{T} \quad (3.7)$$

The sensitivity of the open-loop system to changes in the plant $G(s)$ is equal to 1. The sensitivity of the closed loop is readily obtained by using equation 3.7. The system transfer function of the closed-loop system is:

$$T(s) = \frac{Gc(s)G(s)}{1 + Gc(s)G(s)} \quad (3.5)$$

Therefore the sensitivity of the feedback system is:

$$S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T} = \frac{Gc}{(1 + GcG)^2} \cdot \frac{G}{GGc/(1 + GcG)} \quad (3.8)$$

Or:

$$S_G^T = \frac{1}{1 + Gc(s)G(s)} \quad (3.9)$$

For the open loop in Figure 3.5.a

$$Y(s) = Gc(s)G(s)R(s) + GD(s) \quad (3.10)$$

$$Y_R(s) = Gc(s)G(s)R(s) \quad (3.11)$$

$$Y_D(s) = GD(s) \quad (3.12)$$

For the closed-loop system in Figure 3.5.b, assuming no input signal ($R(s)=0$) and unity feedback, then

$$Y_D(s) = \frac{G}{1 + G_c G} D(s) \quad (3.13)$$

Now assuming no input signal ($D(s)=0$) then

$$Y_R(s) = \frac{Gc(s)G(s)}{1 + Gc(s)G(s)} R(s) \quad (3.14)$$

Then,

$$Y(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}R(s) + \frac{G}{1 + G_c G}D(s) \quad (3.15)$$

Where;

Y_R is the output due to the input signal

Y_D is the output due to the disturbance signal

The ratio $Y_D(s)/Y(s)$ represents the disturbance contribution to the system.

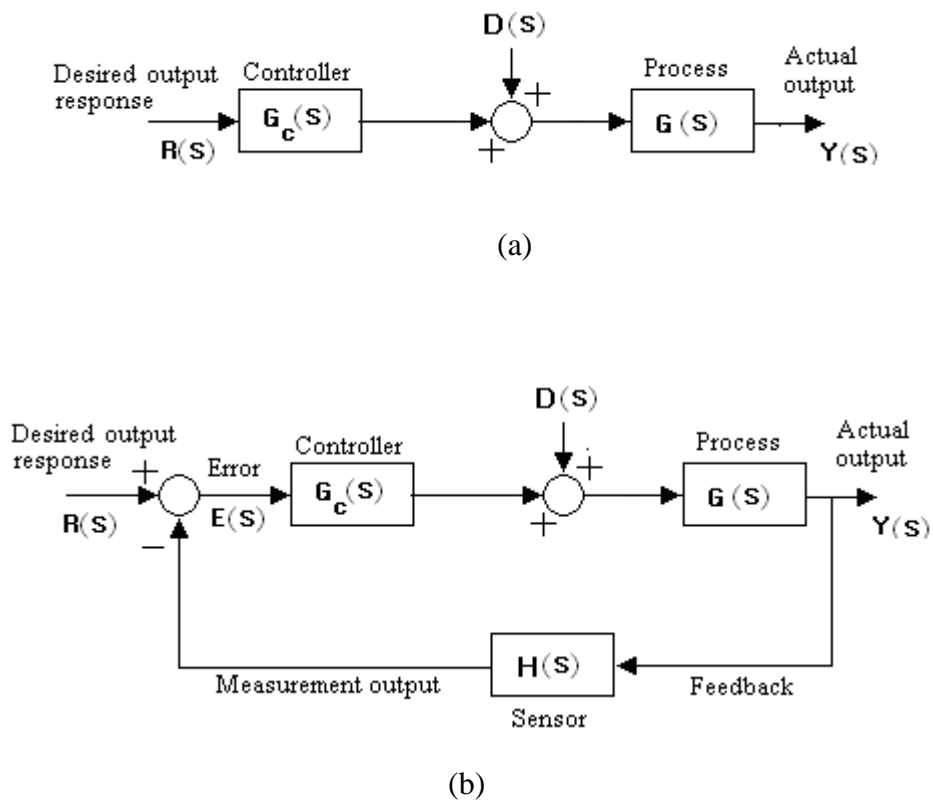


Figure 3.5 (a): Open Loop Control system with Disturbance Signal
(b): Closed-loop Control System with Disturbance Signal

Procedure:

Connect the servomotor system where its input is voltage and its output is speed as shown by Figure 3.6. Don't forget to connect the power supply to the summing amplifier and the tachogenerator LCD.

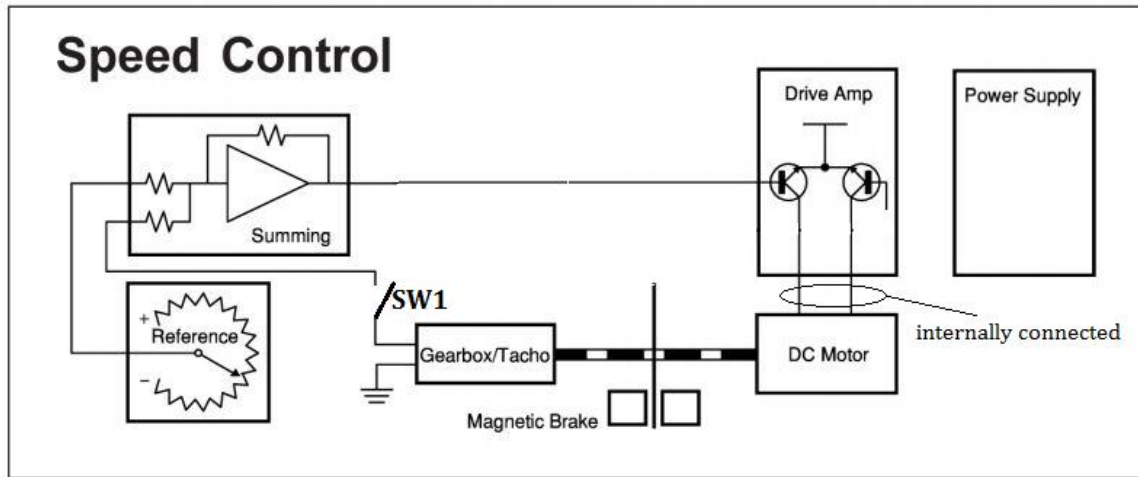


Figure 3.6: Speed Controlled Servomotor

Part 1: Steady state error.

1. Set the reference voltage to 1V and the summing amplifier gain to 1.
2. While the switch SW1 is open and the magnetic break is removed, observe the speed of the motor and record the LCD voltage reading.
3. Increase the gain of the summing amplifier to 2 by changing its feedback resistance. Observe the change in motor speed (increased slightly, increased noticeably, decreased slightly or decreased noticeably) and record the LCD voltage reading.
4. For the same above settings, close the switch SW1, observe the speed of the motor and record the LCD voltage reading.
5. Decrease the gain of the summing amplifier to 1 by changing its feedback resistance. Observe the change in motor speed (increased slightly, increased noticeably, decreased slightly or decreased noticeably) and record the LCD voltage reading.

Part 2: Sensitivity

1. Set the reference voltage to 1V and the summing amplifier gain to 1.
2. While the switch SW1 is closed and the magnetic break is removed, observe the speed of the motor and record the LCD voltage reading.
3. Increase the gain of the summing amplifier to 2 by changing its feedback resistance. Record the LCD voltage reading.
4. Increase the gain of the summing amplifier to 3 by changing its feedback resistance. Record the LCD voltage reading.

Part 3: Disturbance rejection

1. Set the reference voltage to 1V and the summing amplifier gain to 1.
2. While the switch SW1 is open, apply the magnetic break as instructed by your lab supervisor. Observe the speed of the motor and record the LCD voltage reading.
3. For the same above settings, increase the gain of the summing amplifier to 2 by changing its feedback resistance. Observe the change in motor speed (increased slightly, increased noticeably, decreased slightly or decreased noticeably) and record the LCD voltage reading.
4. For the same above settings, close the switch SW1, observe the speed of the motor and record the LCD voltage reading.
5. For the same above settings, decrease the gain of the summing amplifier to 1 by changing its feedback resistance. Observe the change in motor speed (increased slightly, increased noticeably, decreased slightly or decreased noticeably) and record the LCD voltage reading.

Discussion and Analysis:

1. What is the difference between open loop system with magnetic brake and without magnetic brake?
2. What is the difference between closed loop system with magnetic brake and without magnetic brake?

Experiment 4

MATLAB Tutorial

Objectives:

- To familiarize the students with MATLAB software.
- To learn how to represent polynomials in MATLAB, find roots of polynomials and to create polynomials for known roots.
- To find the transfer function using MATLAB and to analyze.
- To find response of transfer function for step input and to analyze the response.

Theoretical Background:

% to enter a polynomial to matlab, use matrices

```
a=[1 2 3]
```

% to convert the polynomial into a string (just to view it but you CAN'T substitute in it)

```
b=poly2str(a,'t')
```

% to substitute in the polynomial a and find a value:

```
polyval(a,3)
```

% to find the roots of the polynomial a:

```
roots(a)
```

% to find the polynomials from the roots:

```
poly(a)
```

% to multiply 2 polynomials a and b (find convolution), use:

```
c=conv(a,b)
```

% if you have a numerater (a) and a denomenator (b), you can find the transfer function $T=a/b$ using:

```
T=tf(a,b) or: T=tf([num parameters], [denum parameters])
```

% if you have a transfer function and you want to have it in the form $T1=Gain*zeroes\backslash poles$, you can use:

$T1=zpk(\text{the name of the original tf})$

% if you have the transfer function's zeroes, poles and gain, and you want to convert it into a transfer function, use:

$T2=zpk([\text{zeroes}],[\text{poles}],\text{gain})$. ex.: $zpk([-1],[-i,i],2)$

% to find the transfer function equivalent to 2 transfer functions T1 and T2 in series, use:

$G=\text{series}(T1,T2)$

% this gives the same result as:

$G=T1*T2$

% to find the transfer function equivalent to 2 transfer functions T1 and T2 in parallel, use:

$G=\text{parallel}(T1,T2)$ % if it is +, or:

$G=\text{parallel}(T1,-T2)$ % if it is -

% this is the same as:

$G=T1+T2$ or

$G=T1-T2$

% to find the transfer function equivalent to a feedback system:

$G=\text{feedback}(T1,T2)$ (by default a – feedback) % or:

$G=\text{feedback}(T1,T2,-1)$ % for - and:

$G=\text{feedback}(T1,T2,+1)$ % for +

% to find the poles of the TF "G":

$\text{pole}(G)$

% to find the zeroes of the TF "G":

$\text{zero}(G)$

% to draw (map) the poles and zeroes of system (Tf "G") in the S plane:

$\text{pzmap}(G)$

% to find the response of a TF "G" to a unity step input:

step(G)

% to find the value of a TF "G" at steady-state "dc-gain", where "dc-gain" is defined as the value of the TF when $s \rightarrow 0$:

dcgain(G)

% to find information about a step response:

stepinfo(G)

% to find a transfer function "G" in its most simple form:

minreal(G)

% to find the natural frequency and the damping ratio for "G":

damp(G)

% to find laplace transform:

symst;

enter the time domain function like:

r=5*t;

laplace(r)

Discussion and Analysis:

Consider the feedback system depicted in the figure shown below.

1. Compute the close-loop transfer function using the series and feedback functions.
2. Obtain the close-loop system unit step response with the step function.
3. Find the final value of the output.

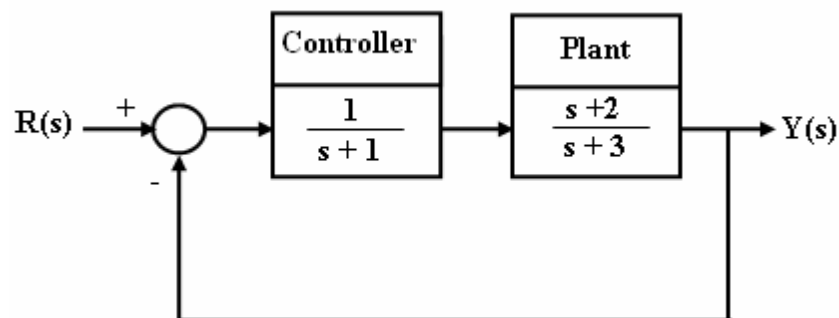


Figure 4.1: System Transfer Function

Experiment 5

Modeling and Simulation

Objectives:

- Learn to obtain the mathematical model of physical systems.
- Learn how to model the system in Simulink.
- Learn how to use Simulink to build the block diagram representing the ODE.
- Learn how to transfer the output signal and other signals to MATLAB workspace.
- To obtain the system response by using Simulink.

Theoretical Background:

Simulink is simulation and link is an extension of MATLAB. It works with MATLAB to offer modeling, analyzing of dynamical systems under a graphical user interface (GUI) environment.

Modeling: is an abstraction or representation of actual real system.

Analyzing: is investigate the performance of a given system.

Modeling the system

Consider a mass-spring-damper system with step input and zero initial conditions as shown in the Figure (5.1), obtain the mathematical modeling of the system.

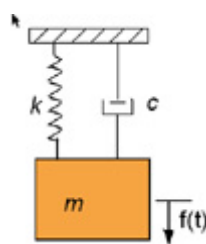


Figure (5.1): a mass-spring-damper system

The mathematical model for this system is described by Equation 5.1.

$$F(t) = kx + b x' + m x'' \quad (5.1)$$

Simulation by using Simulink:

1. Run MATLAB program.
2. In MATLAB command window write simulink then press enter or click on the Simulink icon located on the toolbar as shown in figure 5.2.

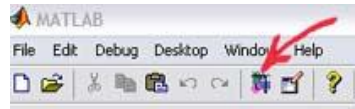


Figure 5.2: Simulink Icon

The Simulink Library Browser window should appear as shown in Figure 5.3.

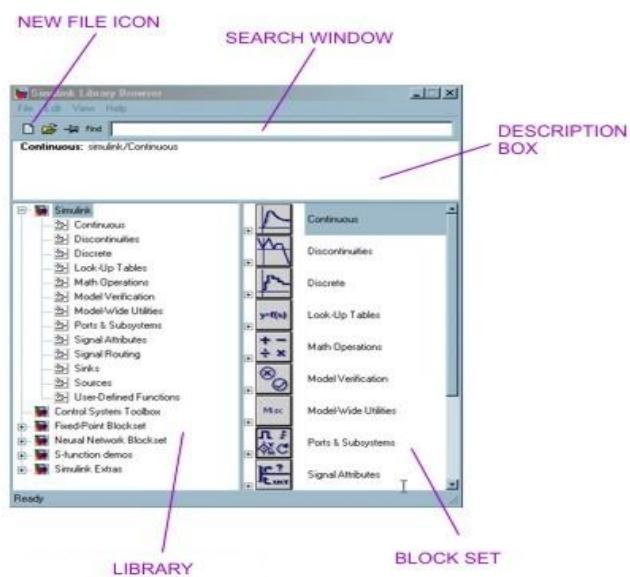


Figure 5.3: Simulink Library Browser window

3. To start a model click on the new file icon as shown in figure 5.3 or you may use keystrokes (CTRL+N). A new window will appear as we see in figure (5.4)

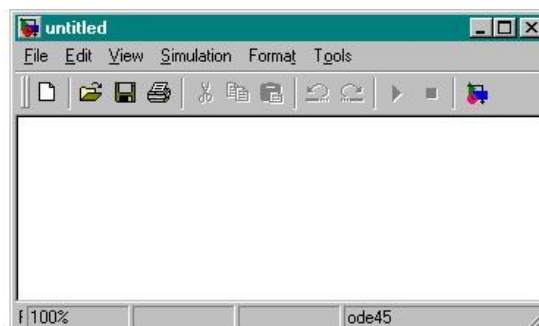


Figure 5.4: New Simulink Window

4. Go to simulink library browser window, Select and drag the required blocks on new window and built the require program, table 5.1 shows the most required blocks which are using in the control systems.

NOTE:

- Use the search window to locate a block knowing its name by enter the name.
- Use CTRL+I to flip and CTRL+R to rotate the blocks if necessary.

Table 5.1: Most required blocks in the control systems

Block	Place	Function
step	source	step input
gain	Math	Gain
sum	Math	Summation point
Transfer Fcn	continuous	Transfer function
scope	Sinks	Oscilloscope
Integrator	Continuous	Integrator
Mux	Signal routing	Multiplexer
To Workspace	Sinks	See the result in matlab workspace

5. To transfer the output signal from the "To Workspace" block sees figure 5.5 in Simulink to MATLAB workspace:

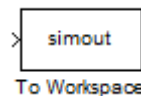



Figure 5.5: "To Workspace" Block

Connect the result with "to workspace" block then double click on that block to bring up the parameter dialog window and change the save format to array.

6. Run the simulation, click Simulation in the tool bar and select Start, or equivalently hit (Ctrl+T) on the keyboard, or click the  button on the tool bar.
7. Save the Simulink file.

Simulation Example:

Consider a mass-spring-damper system with step input and zero initial conditions as shown in the Figure 5.1. That have ($m = 2, c=5, k=3$), Use Simulink to simulate this system.

Step1: re-arrange the Equation 5.1 to obtain Equation 5.2:

$$x'' = 1/m[bx' + kx + f(t)] \tag{5.2}$$

Step 2: open a new simulink window (CTRL+N) and drag the required blocks from the Simulink library window, the simulink program is shown in Figure 5.6.

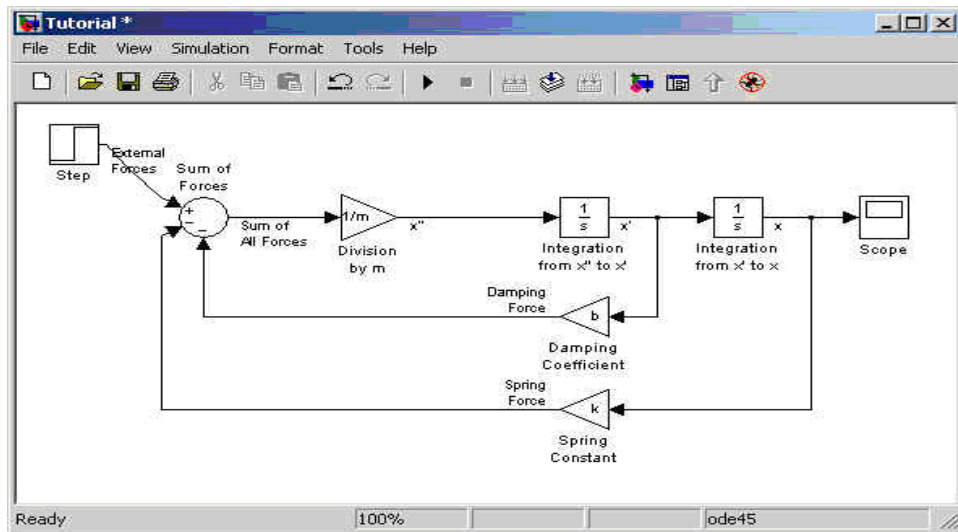


Figure 5.6: The Simulink Program of Mass –Spring –Damper System

Step 3: Enter the values of the parameters for each block. And change the parameters of step source.

Step4: Run the simulation, to view the output of the Scope double click on the Scope in the model. The following plot of the step response will appear in Figure 5.7.

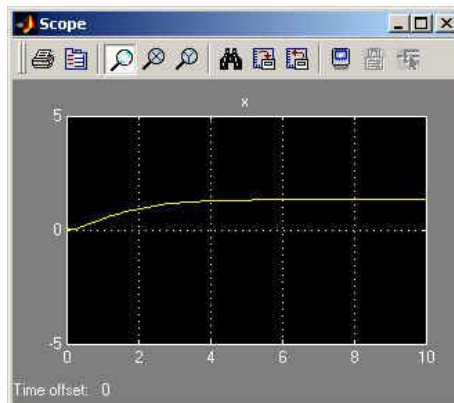
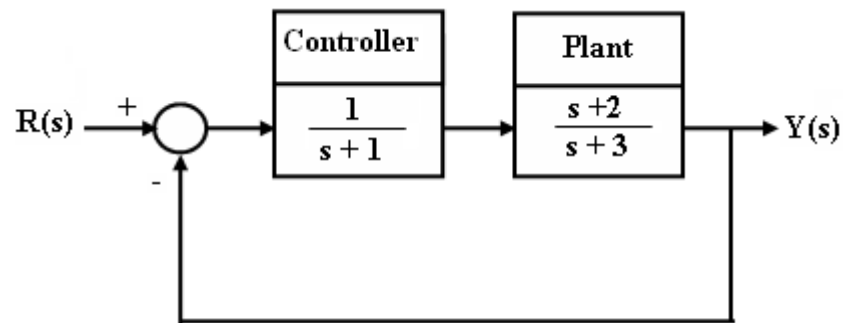


Figure 5.7: Step Response of the System

Discussion and Analysis:

1. Model the system that shown below in Simulink and obtain step response.



Experiment 6

Root Locus Design

Objectives:

The objective of this lab is to introduce controller design based on the root locus method.

Theoretical Background:

The root locus of a system is the plot of the paths (loci) of all possible closed loop poles as the design parameter takes on a range of possible values. Closed-loop response depends on the location of closed-loop poles.

The root locus of an (open-loop) transfer function Plant is a plot of the locations (locus) of all possible closed loop poles with proportional gain k and unity feedback. The command used to plot the RL of the system is (rlocus)

The rltool is a graphical user interface (GUI) tool that allows you to analyze and tune SISO feedback control systems. Using the rltool, you can graphically tune the gains and dynamics of the compensator (C) and prefilter (F) using a mix of root locus and loop shaping techniques. For example, you can use the root locus view to stabilize the feedback loop and enforce some minimum damping, and use the Bode diagrams to adjust the bandwidth, check the gain and phase margins, or add a notch filter for disturbance rejection.

Procedure:

To start with designing using root locus first you must define the transfer function of the system to be controlled on MATLAB work space as follow.

```
>> plant = tf(1, [2 5 3])
```

Transfer function:

$$\frac{1}{2s^2 + 5s + 3}$$

Then use the command (rltool) to open the rltool window shown in Figure 6.1.

```
>> rltool
```

At the top right corner of the window the block diagram of the closed control system is shown, now you need to import the transfer function of your system (open-loop function) and place it in the G block of the block diagram. From file choose import.

From the next window(see Figure 6.2) select the transfer function "**plant**" then click on the arrow next to G. leave the feedback gain H the, prefilter F and the compensator C as default (=1). If you click on the button (Other...) this will allow you to change the architecture of the block diagram. When finish, click the OK button. After that the root locus of your system will appear as in Figure 6.3

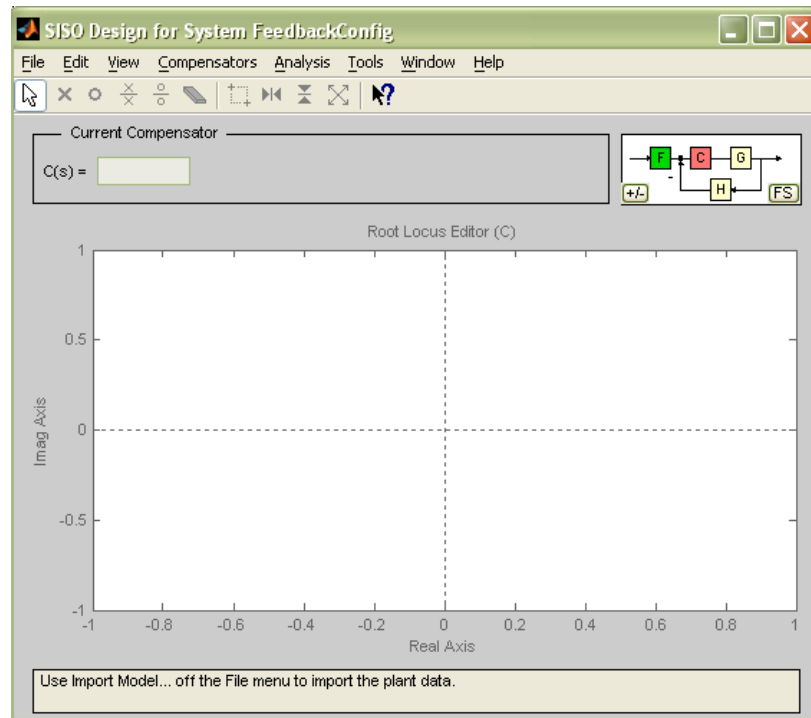


Figure 6.1: SISO Design for System Configuration

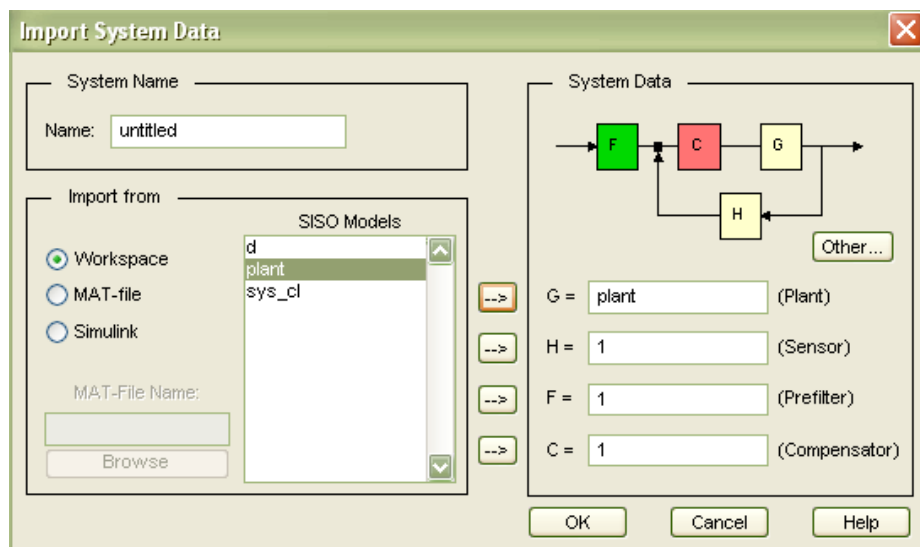


Figure 6.2: Import System Data

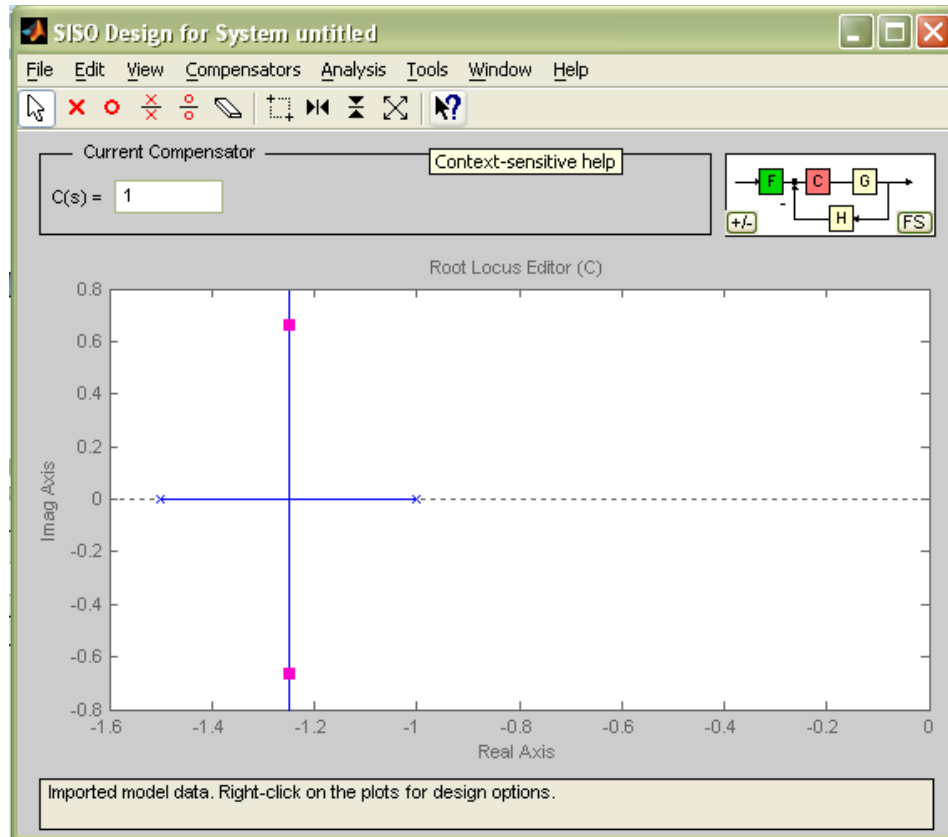


Figure 6.3: The Root Locus of The Transfer Function "plant"

Note the pink squares represent the location of the closed-loop poles at the current gain value. Change the gain of the compensator "C(s)" and observe the new location of the poles.

In this tool box you can add constrains such as overshoot, natural frequency, damping ratio and settling time etc....,

You can also design a compensator by adding poles and/or zeros or even importing a controller transfer function from the workspace. You can retune the gain of your compensator to achieve poles location within the constrains you assigned. You can also see response of the closed-loop system to step input, step disturbance, bode response, open-loop Nyquist, and many other options.

Discussion and Analysis:

For following open loop transfer function, use MATLAB root locus tool to answer the following;

$$T(s) = \frac{s}{s^2 + 2}$$

1. How many poles and zeros does the system have?
2. Use MATLAB to plot the root locus for the system.

Experiment 7 Design and Tuning of PID Controller

Objectives:

- To learn how to design and tune PID controller.

Apparatus:

PID Kit (see Figure 7.1)

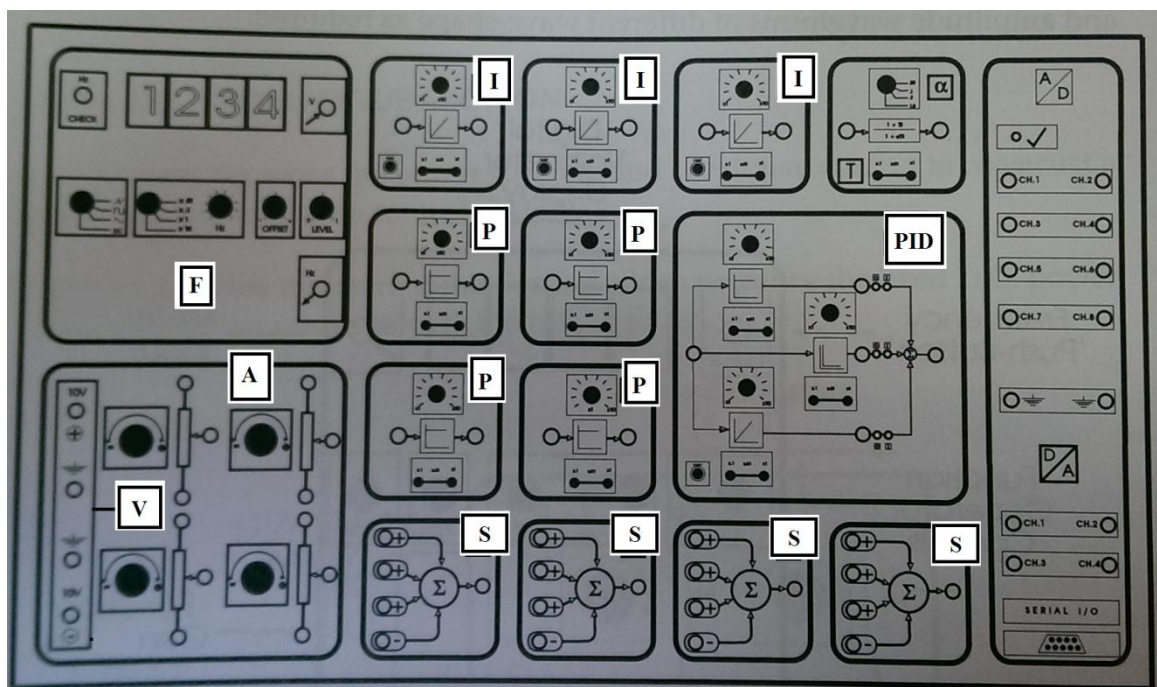


Figure 7.1: PID Kit

Theoretical Background:

You've probably seen the terms defined before: P -Proportional, I - Integral, D - Derivative. These terms describe three basic mathematical functions applied to the error signal, $V_{error} = V_{set} - V_{sensor}$. This error represents the difference between where you want to go (V_{set}), and where you're actually at (V_{sensor}). The controller performs the PID mathematical functions on the error and applies their sum to a process (motor, heater, etc.) So simple, yet so powerful! If tuned correctly, the signal V_{sensor} should move closer to V_{set} .

The PID controller has the following transfer function;

$$PID = k_P + sk_D + \frac{k_I}{s} = \frac{s^2k_D + sk_P + k_I}{s} \quad (1.7)$$

Tuning a system means adjusting three multipliers K_p , K_i and K_d adding in various amounts of these functions to get the system to behave the way you want. Table 7.1 and 7.2 below summarizes the PID terms and their effect on a control system.

Table 7.1

Term	Math Function	Effect on Control System
P Proportional	$K_P \times V_{error}$	Typically the main drive in a control loop, K_P reduces a large part of the overall error.
I Integral	$K_I \times \int V_{error} dt$	Reduces the final error in a system. Summing even a small error over time produces a drive signal large enough to move the system toward a smaller error.
D Derivative	$K_D \times dV_{error} / dt$	Counteracts the K_P and K_I terms when the output changes quickly. This helps reduce overshoot and ringing. It has no effect on final error.

Table 7.2

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
K_p	Decrease	Increase	Small Change	Decrease
K_i	Decrease	Increase	Increase	Eliminate
K_d	Small Change	Decrease	Decrease	Small Change

Note that these correlations may not be exactly accurate, because K_p , K_i , and K_d are dependent on each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for K_i , K_p and K_d .

Implementing the PID Controller Using Op-Amp

Figure 7.2 shows how to separately implement a P, D and an I controllers.

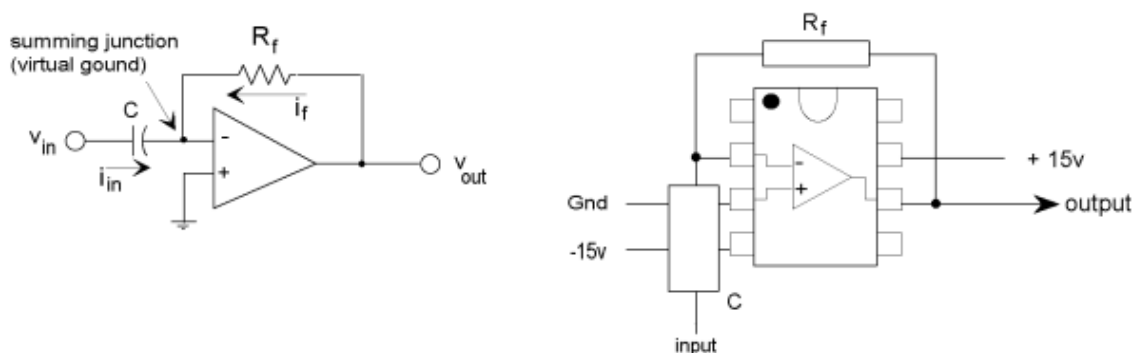


Figure 7.2.a: Derivative Controller

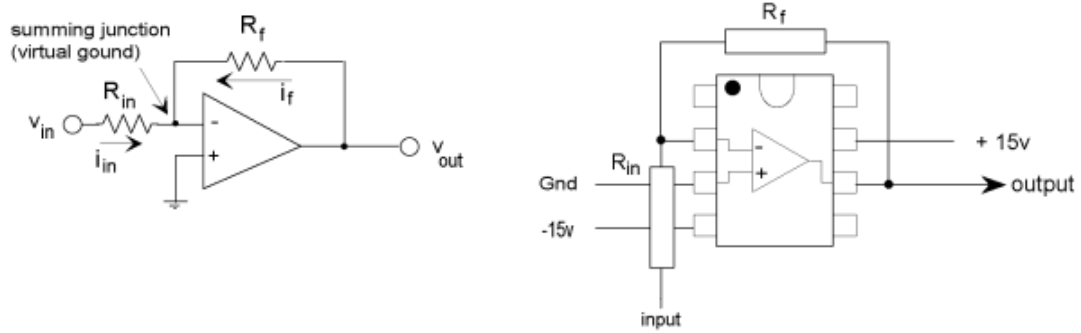


Figure 7.2.b: Proportional Controller

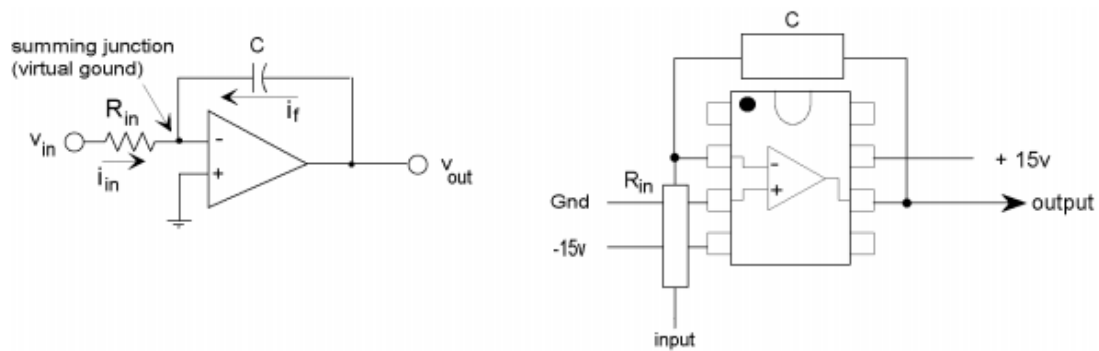


Figure 7.2.c: Integral Controller

Tuning the PID controller:

Tuning the PID controller can be like learning to roller blade, ski or maybe riding a bull. Until you've done it a few times, the literature you've read really doesn't hit home. But after few attempts (and falls), you find it wasn't so bad after all - in fact it was kind of fun!

Although you'll find many methods and theories on tuning a PID, here's a straight forward approach to get you up and solving quickly.

1. SET KP. Starting with $K_P=0$, $K_I=0$ and $K_D=0$, increase K_P until the output starts overshooting and ringing significantly.
2. SET KD. Increase K_D until the overshoot is reduced to an acceptable level.
3. SET KI. Increase K_I until the final error is equal to zero.
4. Keep adjusting the parameters until you reach the desired response.

Procedure:

Implementation of Control Systems Using the PID Kit

Figure 7.1 show the PID kit used in the laboratory. And Table 7.3 explains the uses of each part of this kit.

The best way to explain the PID kit used the laboratory is through an example.

Table 7.3

F	Function generator
V	Dc power supply ($\pm 10V$)
I	Integrator
P	Proportional
PID	Proportional , integral, derivative Controller
S	Summing point
A	Attenuator (Potentiometer)

Example:

Connect the open loop transfer function $T(s)$, then use the PID controller to control the performance of that system?

$$T(s) = \frac{1}{s^2 + 5s + 10}$$

Solution:

$$T(s) = \frac{V_o}{U} = \frac{1}{s^2 + 5s + 10} \dots\dots\dots 7.2$$

Rearranging the equation:

$$V_o'' = U - 5V_o' - 10V_o \dots\dots\dots 7.3$$

The system described by equation 7.3 is represented in block diagram in figure 7.3

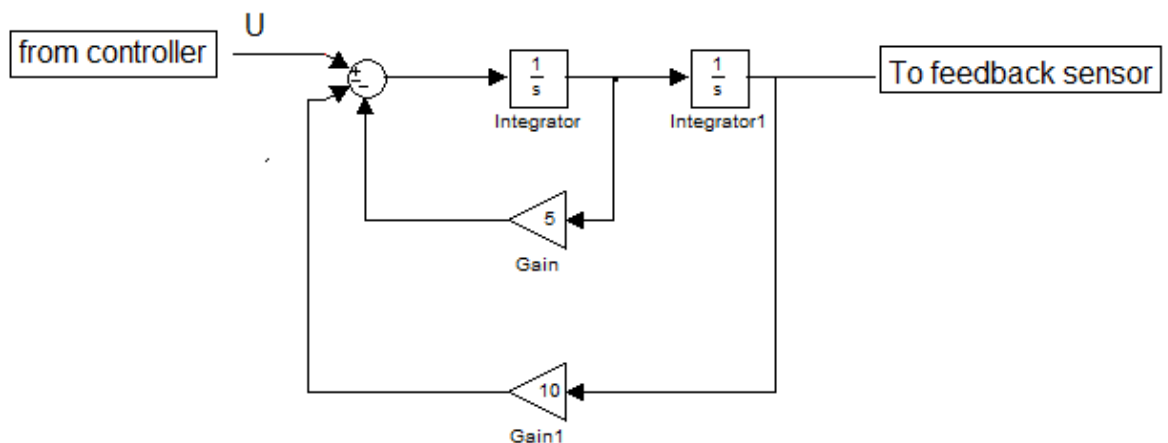


Figure 7.3: Block Diagram for $T(s)$

Now we can build the system using our kit in Figure 7.4

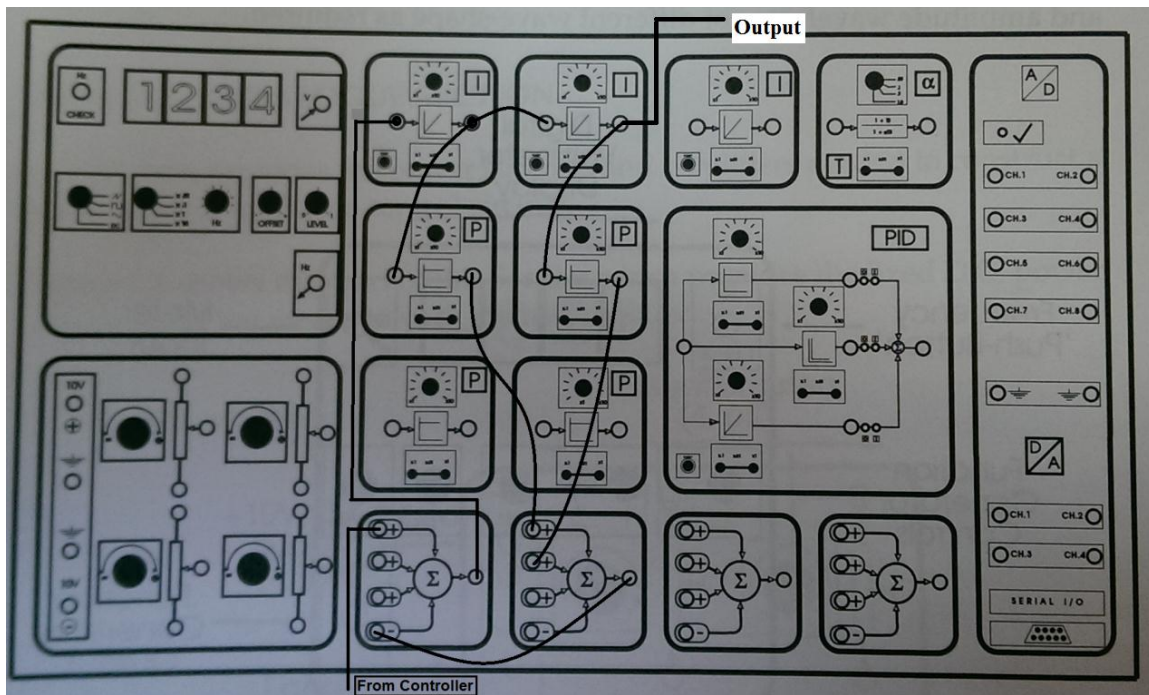


Figure 7.4: Implementation of T(s)

To control the system using a closed loop PID controller complete as in Figure 7.5. The input to the system can be obtained from the function generator or the potentiometer for the PID kit or to any external power supply. Both inputs and output can be connected to an oscilloscope to observe the response of the system.

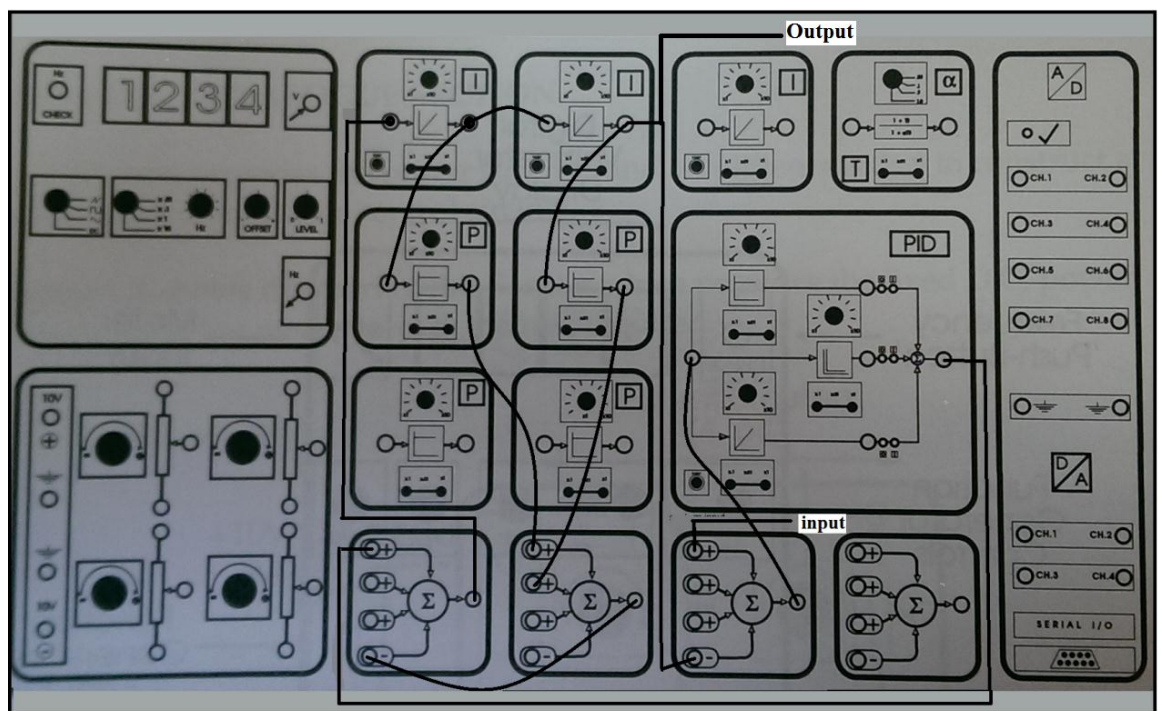


Figure 7.5: Closed Loop PID Control System for T(s)

Discussion and Analysis:

Connect the open loop transfer function $T(s)$, then use the PID controller to control the performance of that system?

$$T(s) = \frac{1}{s^2 + 4}$$

Experiment 8

Operational Amplifier

Objectives:

In this experiment you are expected:

- To experimentally implement circuit with Op-Amps.

Apparatus:

The operational amplifier IC741 is presented in Figure 8.1

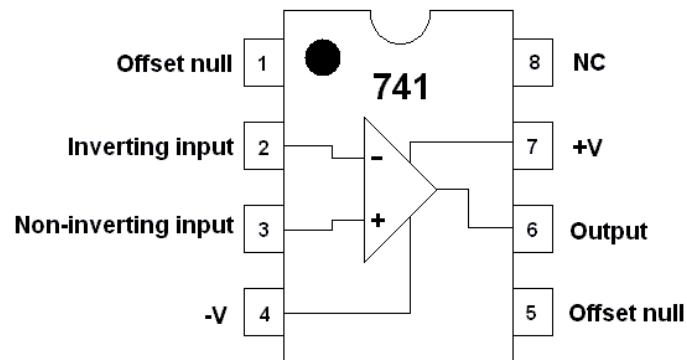


Figure 8.1: The operational amplifier IC741

Theoretical Background:

An operational amplifier (op-amp) is a DC-coupled high-gain electronic voltage amplifier with a differential input and, usually, a single-ended output. An op-amp produces an output voltage that is typically hundreds of thousands times larger than the voltage difference between its input terminals.

The circuit symbol for an op-amp is shown by figure 8.2, where,

V+: non-inverting input

V-: inverting input

Vout: output

Vs+: positive power supply

Vs-: negative power supply

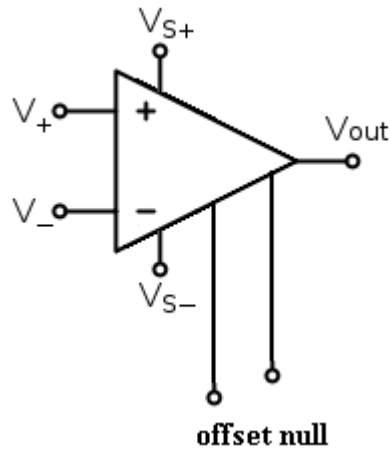


Figure 8.2

Ideal operational amplifier has the following specifications:

1. Infinite input resistance.
2. Zero output resistance.
3. Infinite common mode rejection ratio (CMRR).
4. Zero input current and input voltage offsets.

Ideal op-amp does not really exist. This will cause some practical issues.

Op-amp can be implemented in circuits in different configuration to perform different tasks such as:

1. Inverting amplifier
2. Non-inverting amplifier
3. Summing amplifier.
4. Differential amplifier.
5. Instrumentation amplifier
6. Integration amplifier

Input/output Relation

To derive the input/output relation of an op-amp applies the following three rules:

1. Zero input current ($i_1 = i_2 = 0$)
2. Virtual short between the inverting and non-inverting terminals. ($V_x = V_y$)
3. KCL at each node.

For the circuit in Figure 8.3

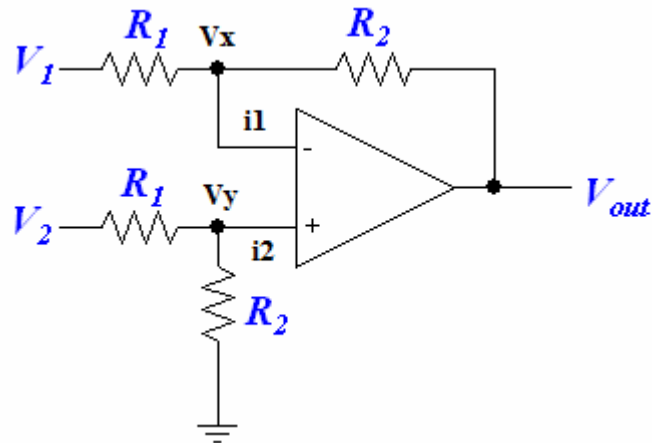


Figure 8.3

KCL at node X

$$\frac{V_x - V_1}{R_1} + \frac{V_x - V_{out}}{R_2} = 0. \Rightarrow$$

$$V_x = \frac{R_2 V_1 + R_1 V_{out}}{R_1 + R_2} \dots\dots\dots 8.1$$

KCL at node Y

$$\frac{V_y - V_2}{R_1} + \frac{V_y}{R_2} = 0. \Rightarrow$$

$$V_y = \frac{R_2}{R_1 + R_2} V_2 \dots\dots\dots 8.2$$

And

$$V_x = V_y \dots\dots\dots 8.3$$

From equations 8.1, 8.2 and 8.3

$$V_{out} = (V_2 - V_1) \frac{R_2}{R_1}$$

Procedure:

Op-Amp Implementation

The pin out of an operational amplifier IC741 is presented in Figure 8.1. For example the schematic circuit in Figure 8.4a is connected using operational amplifier in Figure 8.4b

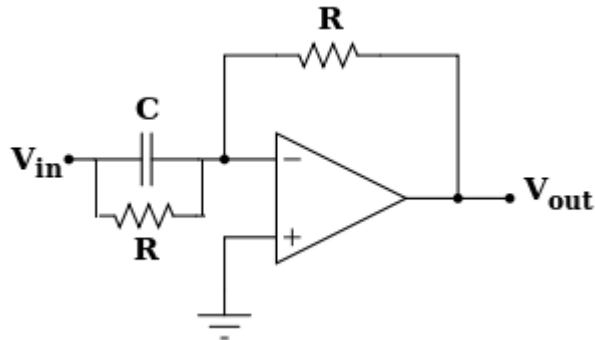


Figure 8.4a

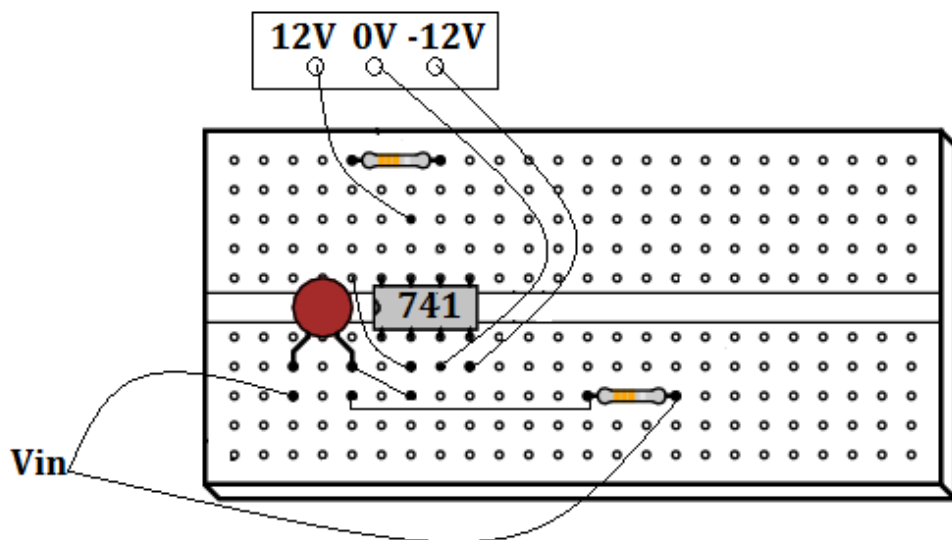


Figure 8.4b

Discussion and Analysis:

Connect the schematic circuit in Figure 8.5 using operational amplifier

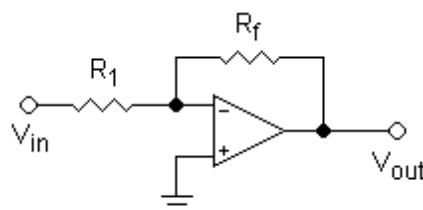


Figure 8.5

Experiment 9

Magnetic Levitation

Objectives:

- To introduce the magnetic levitation system.
- Obtain the system model and linearize it at operation condition.

Apparatus:

The physical system of Magnetic Levitation shown in Figure 9.1.

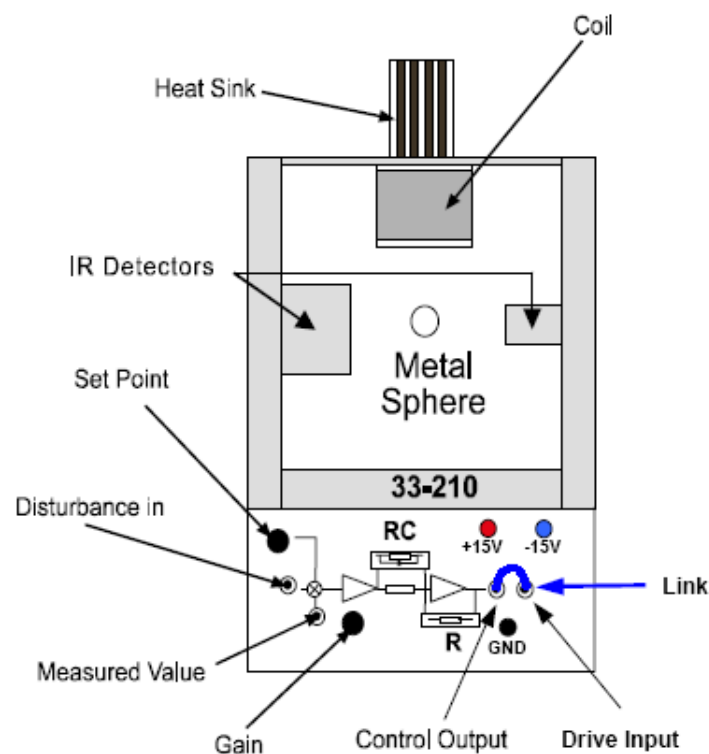


Figure 9.1: Physical System of Magnetic Levitation

Theoretical Background:

The magnetic levitation system has the goal to suspend a steel sphere by means of a magnetic field counter acting the force of gravity, and to be able to apply a controlled disturbance to force the sphere to follow a predetermined trajectory.

The control to be applied is a voltage which is converted into current. This current passes through coil which create magnetic field.

The measure position is determined from an array of infrared transmitters and detectors, positioned such that the infrared beam is intersected by the sphere.

The state of the sphere is described by a single process variable- the vertical position measured by the infrared detectors. Thus the system is Single-Input-Single Output (SISO).

System Modeling:

In order to construct a physical model for the system it is necessary to state the following

- The magnetic field on the axis of the coil is proportional to the current, and the current is proportional to the input control voltage "the voltage to current converter used is proportional".
- The magnetic field induces a dipole in the sphere whose strength is proportional to the magnetic field and hence the current.
- The forces on the sphere are attraction on the upper pole and repulsive on the lower pole. This force is proportional to the strength of the pole and the magnetic field at the location of the pole.

According to that;

$$i = kV_{in} \quad (9.1)$$

$$F_m = K \frac{i^2}{y^2} \quad (9.2)$$

For the free body diagram of the system (neglecting the air friction) the equation of motion is obtain by applying Newton's law in the vertical axes.

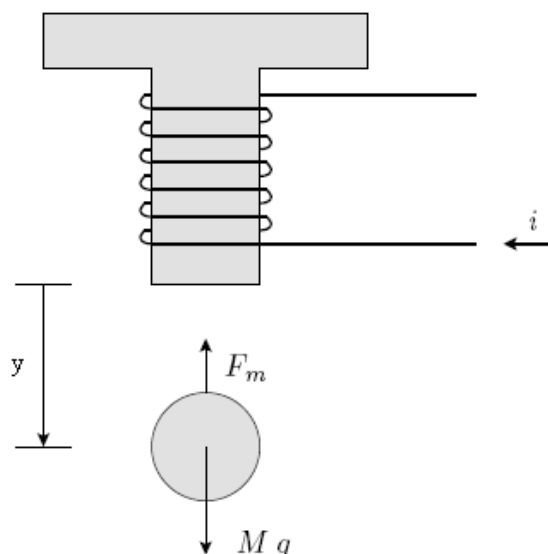


Figure 9.2: Free body diagram

The equation of motion that describes this system is nonlinear due to the presence of the term $\left(\frac{i}{y}\right)^2$.

Linearizing at the operating conditions y_{ss}, i_{ss} using the Taylor expansion:

$$f(x, y) = f(x_0, y_0) + (x - x_0) \left. \frac{\partial f}{\partial x} \right|_{(x_0, y_0)} + (y - y_0) \left. \frac{\partial f}{\partial y} \right|_{(x_0, y_0)} \quad (9.3)$$

Where f is the nonlinear term $\left(\frac{i}{y}\right)^2$, x_0 and y_0 are the operation condition y_{ss}, i_{ss}

Note that at steady state

$$Mg = K \left(\frac{i_{ss}}{y_{ss}} \right)^2 \quad (9.4)$$

Equation 9.5 shows the transfer function of the system

$$T(s) = \frac{Y(s)}{I(s)} = \frac{-a}{s^2 - b} \quad (9.5)$$

Where

$$a = \frac{2K i_{ss}}{M y_{ss}^2} \quad (9.6)$$

and

$$b = \frac{2K i_{ss}^2}{M y_{ss}^3} \quad (9.7)$$

For this system

$i_{ss}=1$ [amp], $y_{ss}=7$ [mm], $K=32654$ [N.mm²/amp²], $M=.068$ [Kg] and $g=9810$ [mm/s²]

Discussion and Analysis:

1. Derive equation 9.5 and give the values of a and b.